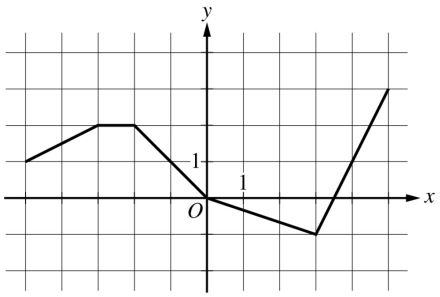
Grading L'Hospital's Rule: (Not on the 2017 US Main, but see the Form I exam to be released via the audit this fall.)

Example: (an "Addendum" to 2017 US Main, AB-6)

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(e*) Find $\lim\limits_{x\to 4} \dfrac{2h(x)-\sqrt{x}}{\sin(\pi x)}$.



Graph of h

(e*) $\lim\limits_{x\to 4}\big(2h(x)-\sqrt{x}\,\big) = 2\cdot 1-\sqrt{4} = 0$. $\lim\limits_{x\to 4}\sin(\pi x) = \sin(4\pi) = 0$. Using L'Hospital's rule, $\lim\limits_{x\to 4}\dfrac{2h(x)-\sqrt{x}}{\sin(\pi x)} = \lim\limits_{x\to 4}\dfrac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}$ $= \dfrac{2\cdot 2-\dfrac{1}{4}}{\pi\cdot 1} = \dfrac{15}{4\pi}$	$2:\begin{cases} 1: \text{ L'Hospital's rule} \\ 1: \text{ answer} \end{cases}$

Notes:

- *L'Hospital's rule [1 pt]*: two requirements
  - indicate $\lim\limits_{x\to 4}\big(2h(x)-\sqrt{x}\,\big) = 0$ and $\lim\limits_{x\to 4}\sin(\pi x) = 0$.
  - clearly attempt derivatives of numerator & denominator: $\dfrac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}$
- *answer [1 pt]*: three requirements
  - correct answer
  - correct derivatives
  - limit notation somewhere in work on ratio of derivatives
- *General*:
  - no limit notation anywhere scores 0/2
  - all notational errors or linkage errors come off the <u>first</u> point

Scoring L'Hospital's Rule...

Response #1: $\begin{array}{l}\big(2h(x)-\sqrt{x}\,\big)\to 0\\ \sin(\pi x)\to 0\end{array}$; $\displaystyle\lim_{x\to 4}\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}=\frac{15}{4\pi}$ Scores: ?-?

Response #2: $\displaystyle\lim_{x\to 4}\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}=\frac{15}{4\pi}$ Scores: ?-?

Response #3: $\displaystyle\lim_{x\to 4}\frac{2h(x)-\sqrt{x}}{\sin(\pi x)}=\frac{0}{0}=\lim_{x\to 4}\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}=\frac{15}{4\pi}$ Scores: ?-?

Response #4: $\displaystyle\frac{2h'(4)-\dfrac{1}{2\sqrt{4}}}{\pi\cos(4\pi)}=\frac{15}{4\pi}$ Scores: ?-?

Response #5: $\displaystyle\lim_{x\to 4}\frac{\dfrac{d}{dx}\big(2h(x)-\sqrt{x}\,\big)}{\dfrac{d}{dx}\big(\sin(\pi x)\big)}=\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\pi\cos(\pi x)}=\frac{15}{4\pi}$ Scores: ?-?

Response #6: $\begin{array}{l}\big(2h(x)-\sqrt{x}\,\big)\to 0\\ \sin(\pi x)\to 0\end{array}$; $\displaystyle\lim_{x\to 4}\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\cos(\pi x)}=\frac{15}{4}$ Scores: ?-?

Response #7: $\displaystyle\lim_{x\to 4}\frac{2h'(x)-\dfrac{1}{2\sqrt{x}}}{\cos(\pi x)}=\frac{15}{4}$ Scores: ?-?

Scoring L'Hospital's Rule...

Response #1... Scores: 1-1

Response #2... Scores: 0-1

Response #3... Scores: 0-1

Response #4... Scores: 0-0

Response #5... Scores: 0-1

Response #6... Scores: 1-0

Response #7... Scores: 0-0