The North Carolina Association Of Advanced Placement® Mathematics Teachers Newsletter

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Volume 19 Winter 2011 Issue No. 1

Web Address
www.ncaapmt.org/calculus

NOTE: Any NC member wishing to be considered for nomination to the Board should contact one of the present board members listed above. Contact Martha Ray at raym@gcsnc.com We presently have openings in the eastern and western regions.

Notes from the President's Desk

As time is drawing near to the completion of teaching the curriculum for your AP Calculus course in the current school year, you may be looking for additional resources to prepare your students for the AP exam. If you are looking for specific questions for additional practice, http://apcentral.collegeboard.com offers a FRQ index to identify questions by function representation as well as a multiple choice index for the released exams. Free Response Questions from 1999 to 2010 with scoring rubrics, commentary, and student samples are available to assist students with their understanding of the scoring process. Sample multiple choice questions can also be found. Dan Kennedy has developed a list of global tips for students along with a list of top ten errors that students make which you might find useful. The course home page offers classroom instruction strategies and resources with lessons and ideas for differentiating instruction. May 4th will be here quickly and we all want our students to be ready for the challenge of the AP exam.
I wish each of you and your students the best on the upcoming exam.

Martha Ray, President
Guilford County Schools, Greensboro, NC

Notes From The Secretary's Desk

I am really busy on several projects. I hope your year is less hectic. Teaching part time has become a full time effort. I am looking forward to attending the AP Reading so that I can have a break. I presently teach 2 classes at Erwin High School in Buncombe County. My hours are from 9:30 - 1:30. Of course that does not include the planning and grading that goes along with teaching. We have had so much snow here that Spring Break is looking less likely and Saturday school has become a reality. There are some useful items in this newsletter from "real" teachers who have used these in their classrooms. I hope you will enjoy and take the time to thank the writers. It would also be great if you sent in some ideas you use with your students.

If anyone is interested in taking over for me as editor of this newsletter, please let me know. I think it is time for new ideas and for someone who might like to design a new format.

Deb Britt, Mars Hill, NC, dgb531@aol.com

Please remember to renew your membership to receive the two yearly newsletters. You can send your $5.00 check, payable to NCA ^{2} PMT, to Jeff Lucia, 718 Lansdowne Road, Charlotte, NC 28270. Email address is jeff.lucia@providenceday.org.

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2011 EXAM CHANGES

There are two major changes coming to the 2011 exam. It is critical that teachers know about these changes. Notices will be sent to teachers and all of this information will be posted on the AP Central website. Please inform your students.

1. **There will no longer be deductions for wrong multiple choice answers. Students should answer all multiple choice questions. This will be true for all AP exams - not just calculus.**

2. **The AP Calculus Free Response (Essay) Section will be divided into 2 questions that are calculator active and 4 that are non-calculator. This is different from the current 3 with a calculator and 3 without.**

CHARBORO BAY OIL LEAK

by Trish Morris (Greensboro Day School) and Rhea Caldwell (Providence Day School)

Part I

Oil is leaking from the Moor Well in Charboro Bay at a rate L(t), where

$$L(t) = \begin{cases} 36, & 0 \le t \le 60 \\ 36 - 0.1(t - 60), & t > 60 \end{cases}$$ in thousands of barrels per day.

f(t) is a continuous function that represents the rate at which oil is recovered from the bay, in thousands of barrels per day, over t days. Selected values of f(t) are presented in the chart below.

t	0	8	24	27	30	48	49	52	58	65
f(t)	0	0.2	1.3	1.9	2.7	12.8	13.5	15.4	18	20.4

1. Approximate the rate of the recovery on the 39^{th} day.

2. State the integral expression to determine the number of thousands of barrels recovered over the time interval $24 \le t \le 52$ days. Use a left hand approximation to evaluate. Indicate units with your answer.

3. It is determined that the algebraic rule for f(t) is given by $R(t) = \dfrac{22}{1 + 350e^{-.129t}}$. R(t) closely models the rate of recovery of the oil, in thousands of barrels per day. Use this algebraic model to determine the approximate number of thousands of barrels recovered over the time interval $24 \le t \le 52$ days.

4. Explain the meaning of $\dfrac{1}{28}\int_{24}^{52} R(t)\,dt$. Indicate units of measure. Find the value of $\dfrac{1}{28}\int_{24}^{52} R(t)\,dt$ using the algebraic rule for R(t) as given in problem #3.

5. What is the meaning of $L(t) - R(t) = 0$? On what day will this occur? Show the reasoning that leads to your answer.

6. For $t > 60$, determine the maximum number of barrels in the bay to the nearest thousand barrels.

7. When is $R''(t) = 0$? What is the significance of this value? Call this time t_1.

8. What are the approximate values of $\int_0^{t_1} R(t)\,dt$, $R(t_1)$, and $R'(t_1)$? Explain the meaning of each of these values.

9. What is the meaning of $L(t) = 0$? Find the value t to make this statement true.

10. At the time found in problem #9, determine, to the nearest thousand, how many barrels of oil remain in the bay?

11. What is value of $\lim_{t\to\infty} R(t)$? What is the meaning of

$\lim_{t\to\infty} R(t)$? When does R(t) essentially equal

$\lim_{t\to\infty} R(t)$?

12. What is the meaning of the statement: $399.514 - \int_{420}^{b} R(t)\,dt = 69.514$, where $b > 420$?

Find the value of b to make this a true statement.

Solutions to CHARBORO BAY OIL LEAK

by Trish Morris (Greensboro Day School) and Rhea Caldwell (Providence Day School)

Part I

Oil is leaking from the Moor Well in Charboro Bay at a rate L(t), where

$$L(t) = \begin{cases} 36, & 0 \le t \le 60 \\ 36 - 0.1(t-60), & t > 60 \end{cases}$$ in thousands of barrels per day.

f(t) is a continuous function that represents the rate at which oil is recovered from the bay, in thousands of barrels per day, over t days. Selected values of f(t) are presented in the chart below.

t	0	8	24	27	30	48	49	52	58	65
f(t)	0	0.2	1.3	1.9	2.7	12.8	13.5	15.4	18	20.4

1. Approximate the rate of the recovery on the 39th day.

$$\frac{f(48)-f(30)}{18} = \frac{12.8-2.7}{18} = \frac{10.1}{18} = .561 \ \frac{\text{thousand barrels}}{\text{day}^2}$$

or approximately 561 barrels per day per day

2. State the integral expression to determine the number of thousands of barrels recovered over the time interval $24 \le t \le 52$ days. Use a left hand approximation to evaluate. Indicate units with your answer.

$\int_{24}^{52} f(t)\,dt \approx$ 3(1.3) + 3(2) + 18(2.6) + 1(7.4) + 3(13.5) = 104.6 thousand barrels or 104,600 barrels

3. It is determined that the algebraic rule for f(t) is given by $R(t) = \frac{22}{1+350e^{-.129t}}$. R(t) closely models the rate of recovery of the oil, in thousands of barrels per day. Use this algebraic model to determine the approximate number of thousands of barrels recovered over the time interval $24 \le t \le 52$ days.

$\int_{24}^{52} R(t)\,dt \approx 195.213$ thousand barrels or 195,213 barrels

4. Explain the meaning of $\frac{1}{28}\int_{24}^{52} R(t)\,dt$. Indicate units of measure. Find the value of $\frac{1}{28}\int_{24}^{52} R(t)\,dt$ using the algebraic rule for R(t) as given in problem #3.

$\frac{1}{28}\int_{24}^{52} R(t)\,dt$ is the average number of barrels per day recovered over the 28 day interval $24 \le t \le 52$.

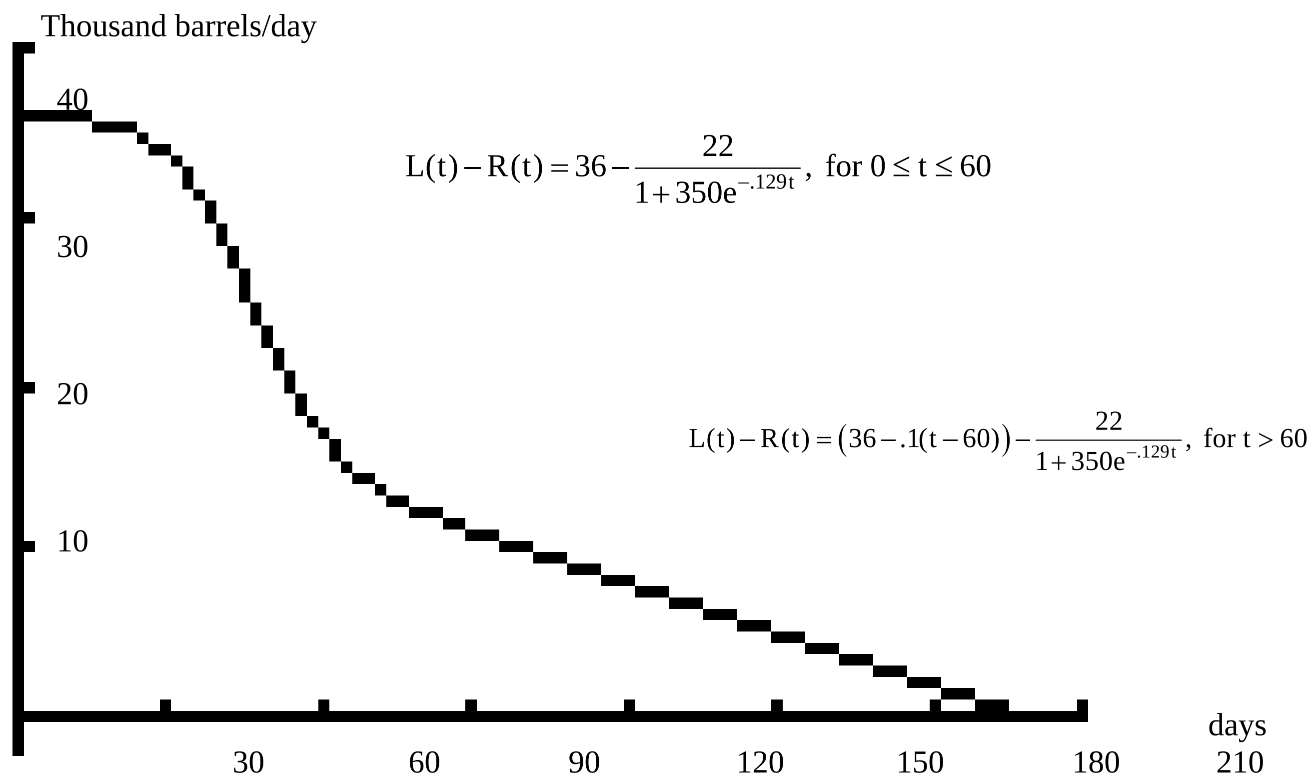
$\frac{1}{28}\int_{24}^{52} R(t)\,dt = 6.972$ thousand barrels per day or 6,972 barrels

5. What is the meaning of $L(t) - R(t) = 0$? On what day will this occur? Show the reasoning that leads to your answer.

$L(t) - R(t) = 0$ means that the rate of recovery equals the rate of the leaking oil. This occurs at t = 200.

For $0 \le t \le 60$, $L(t) - R(t) = 36 - \left(\frac{22}{1+350e^{-.129t}}\right) = 0 \Rightarrow$ no value of t satisfies this statement

For $t > 60$, $L(t) - R(t) = \left(36 - .1(t-60)\right) - \left(\frac{22}{1+350e^{-.129t}}\right) = 0 \Rightarrow t = 200$



6. For $t > 60$, determine the maximum number of barrels in the bay to the nearest thousand barrels.

The maximum number of barrels in the bay to the nearest thousand barrels occurs when
$L(t) - R(t)$ changes from positive to negative. As found in problem #5, this happens when $t = 200$ days. Thus, the approximate number of barrels to the nearest thousand at $t = 200$ is given by

$$\int_0^{60}\left[36 - R(t)\right]dt + \int_{60}^{200}\left[36 - 0.1(t-60) - R(t)\right]dt$$

$\left(36(60) - 344.658\right) + \left(4060 - 3055.828\right) = 2819.514$ thousands barrels or 2,819,514 barrels

7. When is $R''(t) = 0$? What is the significance of this value? Call this time t_1.

$R''(t) = 0$ at $t_1 = 45.410.$ At this time the rate of change in R, $R'(t),$ is a maximum because $R''(t)$ changes from positive to negative (the graph of R changes from concave up to concave down at $t_1 = 45.410$). In real life this means that the rate of change in the rate of removal of the oil, in thousands of barrels per day, is the fastest at $t_1 = 45.410$ days.

8. What are the approximate values of $\int_0^{t_1} R(t)\,dt,\ R(t_1),$ and $R'(t_1)$? Explain the meaning of each of these values.

$\int_0^{t_1} R(t)\,dt = 117.721$ thousand of barrels or 117,721 barrels recovered over

$0 \le t \le 45.410$ days.

$R(t_1) = 11$ thousands of barrels per day or 11,000 barrels recovered per day on day t_1.

$R'(t_1) = .7095$ thousand barrels / day^2 is the rate of change in the recovery rate.
Since $R'(t_1) > 0$ the rate of recovery is increasing.

9. What is the meaning of $L(t) = 0$? Find the value t to make this statement true.

$L(t) = 0$ occurs when the leak is capped. $L(t) = 0$ at $t = 420$ days

10. At the time found in problem #9, determine, to the nearest thousand, how many barrels of oil remain in the bay?

$$\int_0^{60}\left[36-R(t)\right]dt+\int_{60}^{420}\left[36-0.1(t-60)-R(t)\right]dt$$

$\left(36(60)-344.658\right)+\left(-1415.828\right)=$ 399.514 thousand barrels or 399,514 barrels

11. What is value of $\lim\limits_{t\to\infty}R(t)$? What is the meaning of $\lim\limits_{t\to\infty}R(t)$? When does R(t) essentially equal $\lim\limits_{t\to\infty}R(t)$?

$\lim\limits_{t\to\infty}R(t)=22$ is the terminal rate of removal of oil in thousands of barrels per day.

Using the table feature of your calculator, using one decimal place of accuracy, R(t) = 22 at t = 93 days, or using three decimal places of accuracy, R(t) = 22 at t = 129.

X	Y1	
90	21.93	
91	21.939	
92	21.946	
93	21.953	
94	21.958	
95	21.963	
96	21.968	

Y1=21.9526495523

X	Y1	
124	21.999	
125	21.999	
126	21.999	
127	21.999	
128	21.999	
129	22	
130	22	

Y1=21.9995435563

12. What is the meaning of the statement: $399.514-\int_{420}^{b}R(t)\,dt=69.514,$ where b > 420 ? Find the value of b to make this a true statement.

For b > 420 the statement: $399.514-\int_{420}^{b}R(t)\,dt=69.514,$ represents the oil remaining in the bay b days after the leak stared. If $\lim\limits_{t\to\infty}R(t)=22,$ then $\frac{330}{22}\approx 15$ more days. Therefore

b = 420 + 15 = 435 days.

Accumulation: Thoughts On $f(t) = f(a) + \int_a^t f'(x)\,dx$

Lin McMullin, National Math and Science Initiative

The goals of the AP Calculus program include the statement, "Students should understand the definite integral … as the net accumulation of change…."[1] The Topical Outline includes the topic the "definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a).\text{"}^{2}$$

The equation above is simply the Fundamental Theorem of Calculus, but the topic stresses what it means rather than its use as a technique for evaluating a definite integral. Our equation,

$$f(t) = f(a) + \int_a^t f'(x)\,dx,$$

is the FTC expressed in a slightly different form with a change of variables. It becomes a function defined using an integral. The idea and the use of this equation in this form make a myriad of problems easy to understand and work. Yet the word "accumulation" has, to the best of my recollection, never appeared on the exam in this technical sense, nor does it appear in most popular textbooks.[3] [4]

While there are earlier exams[5] on which this idea can be used, the 2000 exam questions AB2/BC2 (d) and especially AB 4[6] is when this approach made its debut. It can be used on many free-response questions since 2000 and in the multiple-choice questions from

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[1] AP Calculus Course Description, The College Board, © 2009, p.7

[2] Ibid. p. 9

[3] This is probably why it is not used on the exams.

[4] "Accumulation" is not mentioned in the indexes of Anton, Finney, Foerster, Hughes-Hallett, Ostebee, Rogawski or Stewart. There is a single sentence in the current edition of Larson, with no exercises using the idea.

[5] For example, 1997 BC 89, see below.

[6] This is a good problem to study. Both the "old" method (solving an initial value differential equation method) and the "new" approach discussed here are shown on the scoring standard. In similar questions in later year only the "new" approach is shown.

the 2003 and 2008 released exams. As with some other topics that appear on the AP calculus exams, since the topic does not appear in the current editions of most of the textbooks used in AP Calculus courses it is important that you use examples from recent exams and make up problems of your own if necessary.

The equation $f(x) = f(a) + \int_a^x f'(t)\,dt$ is, of course, only a simple restatement of the Fundamental Theorem of Calculus. Yet in this form there are a myriad of uses for the equation. On the 2008 AP Calculus exams there are no less than 7 questions where this can be used, yet, as mentioned above, this form and this approach is not mentioned in the most textbooks. It appears 10 times on the 2010 AB scoring standards.[7] We will consider some of the uses of this equation, with some examples from the 2008 and 2009 AB Calculus exams shortly.

The equation says that the function value is equal to some starting value (which may be zero) plus the accumulated change.

Final Value = Starting Value + Accumulated Change

$$f(t) = f(a) + \int_a^t f'(x)\,dx$$

The integral $\int_a^t f'(x)\,dx$ gives the accumulated change (or the net change) from some starting point at *a* to *t* of a function, *f*, in terms of its derivative, f'. When we add the starting value, $f(a)$ to the integral we have the value of the function at *t*.

If the derivative is the velocity, $v(t)$, of a moving object then the integral gives the *displacement* over the time interval [*a*, *t*]. Here the equation gives the position, $s(t)$, of the moving object at time *t*. The object starts at some position $s(a)$.

[7] 2010 A 1(c) is a bit of overkill; the question is at best an Algebra 2 question. However, it is a good, easy, example of the idea.

The equation then looks like this:

$$s(t) = s(a) + \int_a^t v(x)\,dx$$

The first time you saw this equation was for a functions whose rate of change (derivative) was a constant, usually called the slope, *m*. If you know one point of the function, say (x_0, y_0), then the *y*-value anywhere on the function is

$$y(x) = y_0 + \int_{x_0}^{x} m\,dt = y_0 + mt\Big|_{x_0}^{x} = y_0 + mx - mx_0$$
$$y = y_0 + m(x - x_0)$$

This is, of course, the *point-slope equation* of a line. You certainly didn't use the integral in Algebra 1, but the point-slope form is just a special case of our equation. The final value, *y*, is equal to the starting value, y_0, plus the accumulated change. The accumulated change, the change in the *x*-values, is the rate of change, $m = \dfrac{\text{change in } y}{\text{change in } x}$ multiplied by, the actual change in *x*, $x - x_0$.

In calculus we deal with variable slopes, variable rates of change. Here are two similar examples from the 2008 AB Exam. In both the derivative is called the velocity of a moving object.

- 2008 AB 7: A particle moves along the *x*-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position *x* = 2 at time *t* = 0, what is the position of the particle at time $t = 1$? (Multiple-choice answers omitted.)

  Using our equation, the solution can be written immediately:

  $$x(1) = 2 + \int_0^1 3t^2 + 6t\,dt = 2 + \left(t^3 + 3t^2\right)\Big|_0^1 = 2 + 4 = 6$$

- <u>2008 AB 87</u>: An object traveling in a straight line has position $x(t)$ at time *t*. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time *t* = 3? (Multiple-choice answers omitted.)

  The wording is almost identical to the previous example as is the solution. The integration is done by calculator.

$$x(3)=2+\int_0^3\sqrt[3]{1+t^2}\,dt\approx 2+4.51153\approx 6.512$$

These two could have been approached as initial value problems: $\frac{dy}{dx}=f'(x)$ with the initial condition $(a,f(a))$. The "old" approach is to find an antiderivative including a constant *C*, use the initial condition to evaluate *C*, write the particular solution, and finally evaluate the solution at *x* = *a*. The "new" approach, which can be used with any first-order differential equation where the derivative is a function of only one variable, is to use the equation.

$$y(x)=f(a)+\int_a^x\left(\tfrac{dy}{dt}\right)dt=f(a)+\int_a^x f'(t)\,dt$$

Here's a similar question from a long time ago.

- <u>1997 BC 89</u> If f is an antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1)=0$, then $f(4)=$

  (Multiple-choice answers omitted.) Once again the integration is done by calculator especially since the antiderivative is very difficult to compute.

$$f(4)=f(1)+\int_1^4\frac{x^2}{1+x^5}dx\approx 0.375$$

Here is another example from the 2008 AB exam that forces students to recognize this form with a generic function

- <u>2008 AB 81</u>: If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$ (B) $-7+f'(4)$ (C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 \left(-7+f(t)\right)dt$ (E) $-7+\int_2^4 f(t)\,dt$

Answer (E)

On the 2008 AP Calculus tests the equation could be used on all of these questions:

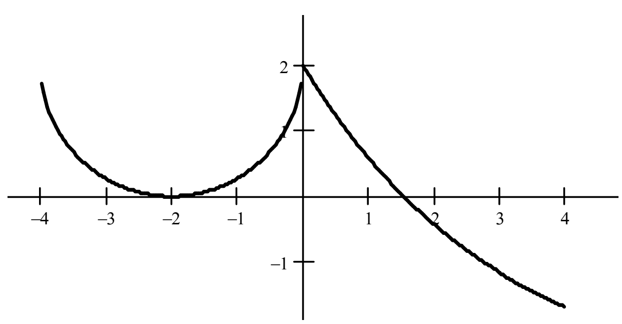
- AB Multiple-choice 7, 81 and 87
- Free-response: AB 2 / BC 2 (d), AB 3 (c), AB 4 / BC 4 (a) *twice* and BC 5 (c).

While other approaches may be possible, any time you are given a starting value and a derivative (rate of change, slope, etc.) this method will get you the answer quickly. Free-response questions often can be solved using our equation. Here is a typical problem, one of many.

<u>2009 AB 6</u> gave the derivative of a function *f* as the piecewise defined function

$$f'(x)=\begin{cases} g(x) & \text{for } -4\le x\le 0 \\ 5e^{-x/3}-3 & \text{for } 0<x\le 4 \end{cases}$$

where $g(x)$ was the semicircle shown on the graph below. The *x*-intercepts of f' are x = –2 and $x=3\ln\left(\frac{5}{3}\right)$. The initial condition is $f(0)=5$(. (All given).



In part (b) students were asked to find $f(4)$ and $f(-4)$. Using our approach the results are straight forward:

$$f(4)=5+\int_0^4 5e^{-x/3}-3\,dx=5+\left(-15e^{-x/3}-3x\right)\Big|_0^4=8-15e^{-4/3}$$

$$f(-4) = f(0) + \int_0^{-4} g(x)\,dx = f(0) - \int_{-4}^{0} g(x)\,dx = 5 - (8 - 2\pi) = 2\pi - 3$$

The second definite integral is most easily found by subtracting the area of the semicircle, 2π, from the area of a rectangle, 8, drawn around it. Notice that $\int_0^{-4} g(x)\,dx$ is "backwards": the upper limit of integration is less than the lower limit. Therefore, its value is the opposite of the area of the region between the semicircle and the *x*-axis.

Integration questions where the lower limit of integration is greater than the upper limit and where the values must be found from a graph are confusing for students. A way of looking at this situation is to reason this way: We are looking for $f(-4)$. If we integrate starting at $x = -4$ we have accumulated the amount $(8 - 2\pi)$ by the time we get to $x = 0$ where the value is 5 (given). So if we *subtract* the $(8 - 2\pi)$ from 5, we will have our starting value $f(-4)$. Symbolically, the details look like this and this avoids the integral with the lower limit greater than the upper.

$$f(-4) + \int_{-4}^{0} g(x)\,dx = f(0)$$

$$f(-4) = f(0) - \int_{-4}^{0} g(x)\,dx$$

$$f(-4) = 5 - (8 - 2\pi)$$

Part (c) of this same question asked students to find the *x* value at which *f* has its absolute maximum. The maximum occurs at $x = 3\ln\left(\frac{5}{3}\right)$ since this is the only place where the derivative of *f* changes from positive to negative. While the preceding sentence receives full credit, here is another approach using the Candidate's test and our equation:

> The candidates for the location of the maximum are $x = -4$, $x = 3\ln\left(\frac{5}{3}\right) = M$, and $x = 4$.
>
> $f(M) = f(-4) + \int_{-4}^{M} f'(x)\,dx$. Since $f'(x) \ge 0$ on the interval $[-4, M]$ the integral is positive, it follows that $f(M) > f(-4)$

$f(4) = f(M) + \int_M^4 5e^{-x/3} - 3\,dx$. Since on the interval [*M*, 4] the integral is negative, it follows that $f(M) > f(4)$

Therefore, the maximum occurs at *x* = *M*.

Admittedly, this is overkill, but it shows another use of the concept. And a few students did use this approach on the exam.

If you look through the AP Calculus exams, especially from 2000 on, you will find many examples where this equation can be used to advantage; if you look through the textbooks you will find no such examples. While there are always other ways to approach these problems, this one is straightforward, easy to understand and applicable in a variety of situations. You need to supplement your textbook so that students will understand and be able to use this approach.

AP Calculus EXAM Format 2011

Section I Multiple Choice **No penalty for guessing - answer every question**

55 minutes Section I, Part A Multiple Choice – without calculator 28 questions

50 minutes Section I, Part B Multiple Choice – with calculator 17 questions

TOTAL ***Section I*** 105 minutes 45 questions @ 1.2 points = 54 points

Short Break for snack, restroom, etc.

Section II Free Response (Essay)

30 minutes Section II 2 essay questions with calculator AB1, AB2

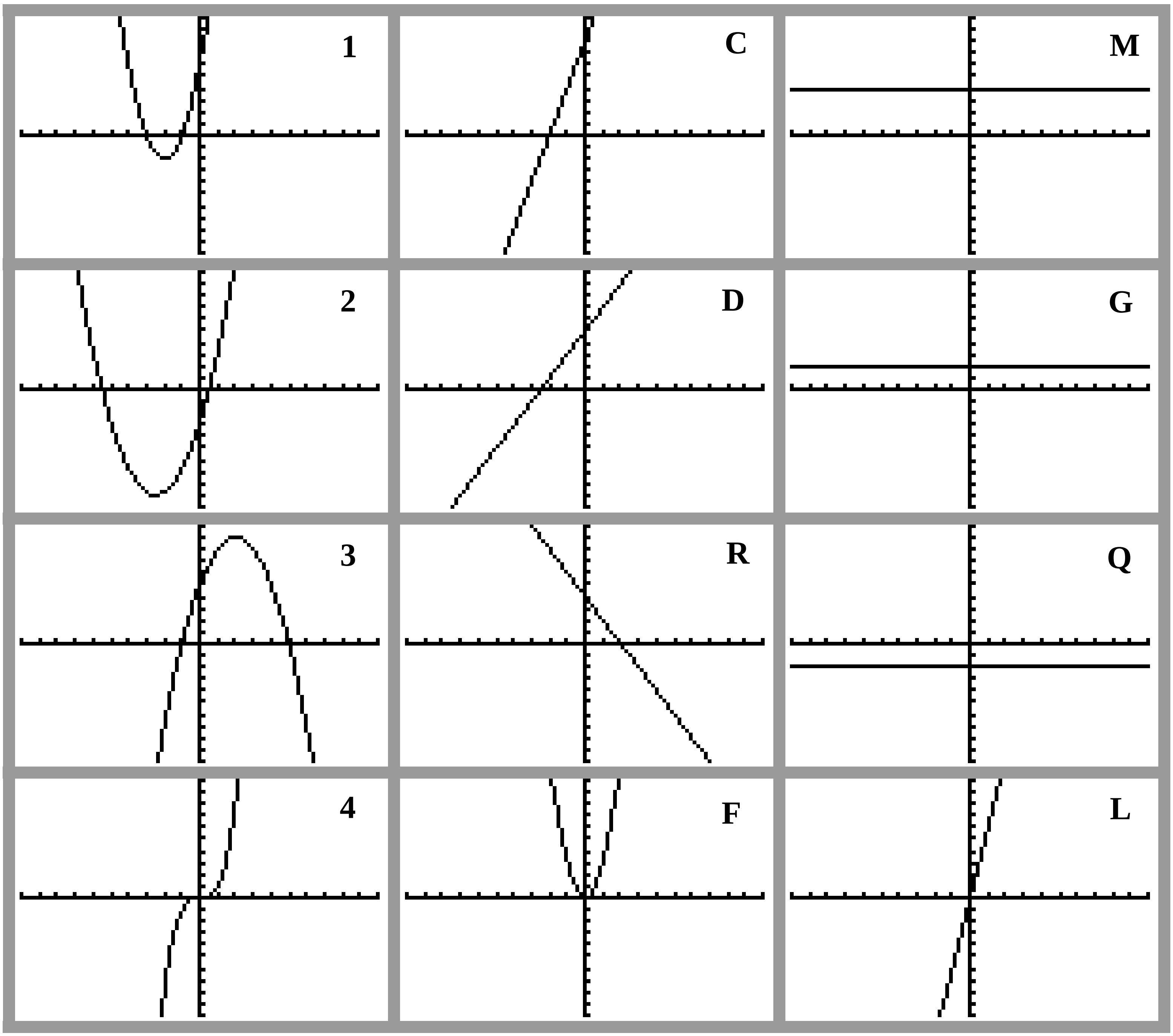
60 minutes Section II 4 essay questions without calculator AB 3, AB4, AB5, AB6 and you can continue to work on AB1, AB2 without a calculator

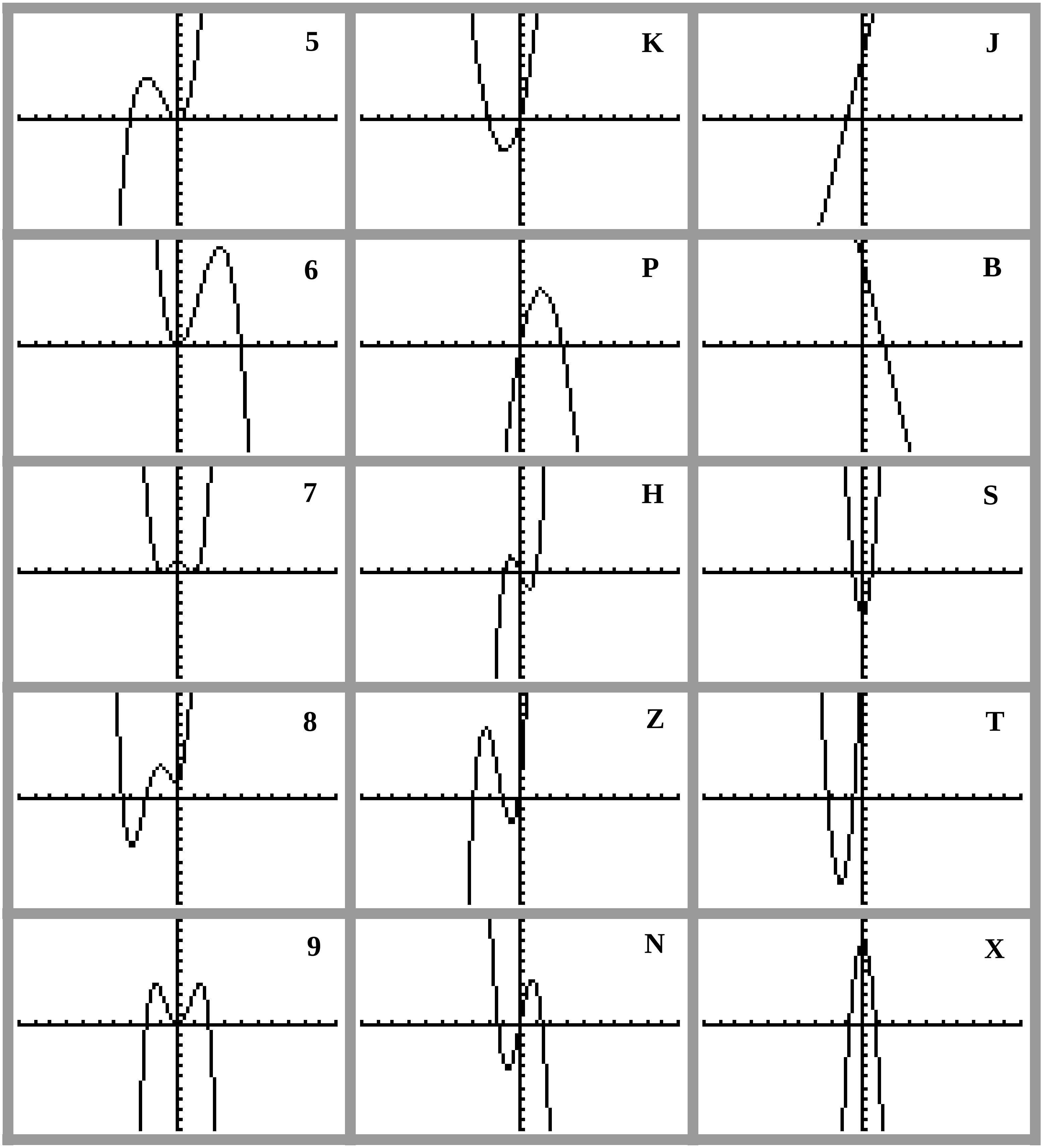
TOTAL ***Section II*** 90 minutes 6 questions @ 9 points = 54 points

EXAM TOTAL 195 minutes = 3 hours, 15 minutes 108 points

Group Graph Activity (Karen Collins, Johnston County)

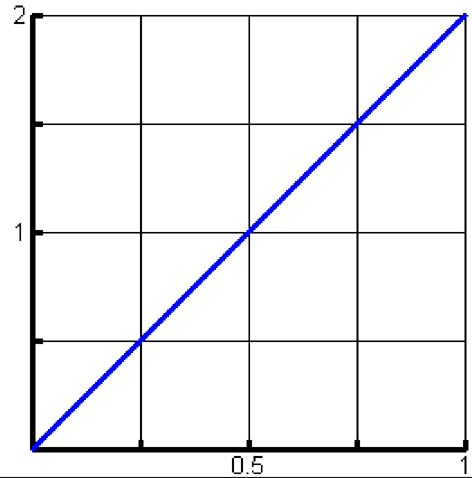
Original functions are numbered 1 through 9.
First and Second derivatives are lettered.
Match each function to its first and second derivative.




5
K
J
6
P
B
7
H
S
8
Z
T
9
N
X

AP Calculus AB **Riemann Sums** **Karen Collins**

1) Find the area under the curve $f(x) = 2x$ in the interval [0, 1] using geometry.



2) Find the area under the curve $f(x) = 2x$ in the interval [0, 1] using **5** partitions.	**4)** Find the area under the curve $f(x) = 2x$ in the interval [0, 1] using **100** partitions.
Find Δx	Find Δx
Find x_i	Find x_i
Find $f(x_i)$	Find $f(x_i)$
Set up the Sigma Notation and compute the value.	Set up the Sigma Notation and compute the value.

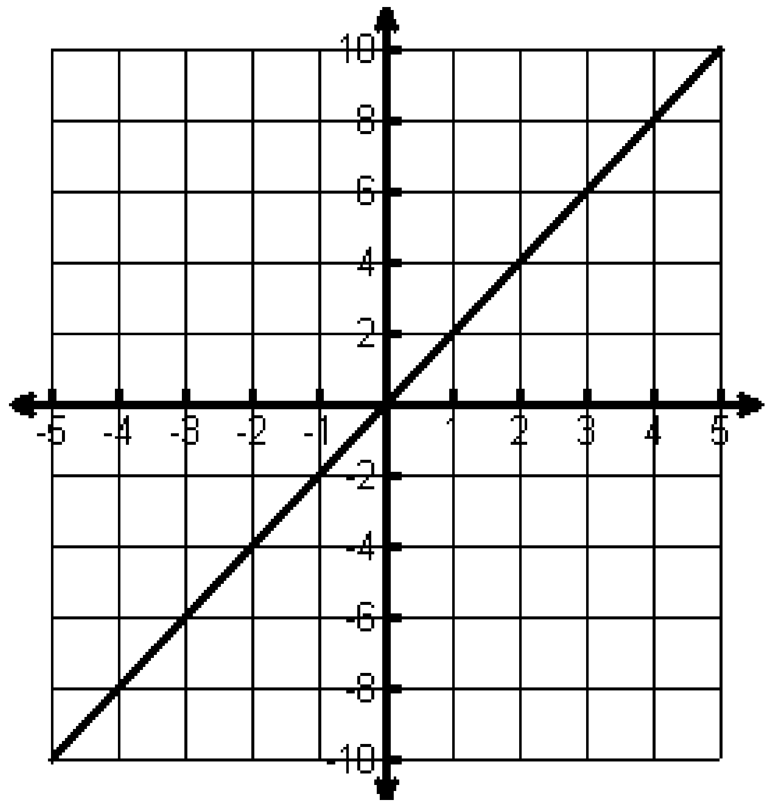
3) Find the area under the curve $f(x) = 2x$ in the interval [0, 1] using **10** partitions.	**5)** Find the area under the curve $f(x) = 2x$ in the interval [0, 1] using ***n*** partitions.
Find Δx	Find Δx
Find x_i	Find x_i
Find $f(x_i)$	Find $f(x_i)$
Set up the Sigma Notation and compute the value.	Set up the Sigma Notation and compute the value.

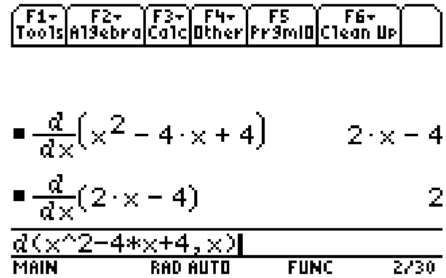
The Antiderivative Connection **(Karen Collins, Johnston County)**

Find the area between the curve and the *x*-axis for $\int_{-2}^{b} 2x\,dx$ where ***b*** represents the upper limit as indicated in the table.

When the table is complete, plot the ***b*** column as *x* and the **Area** column as *y* on the grid below. Write the equation of the graph you plotted.

b	**Area**
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

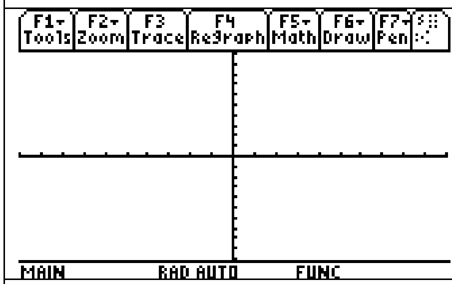
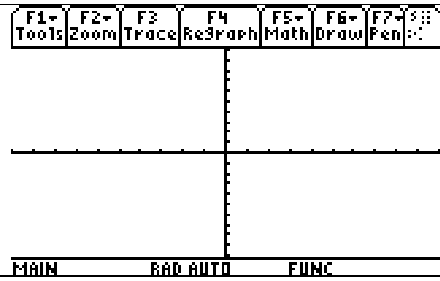
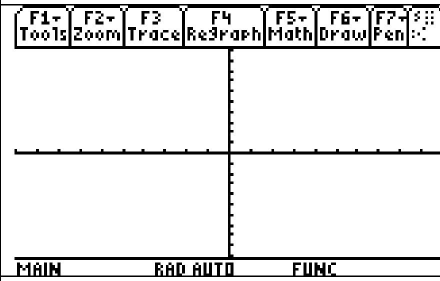
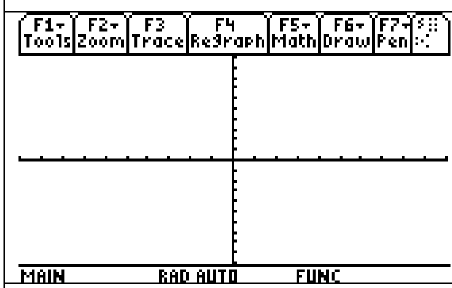
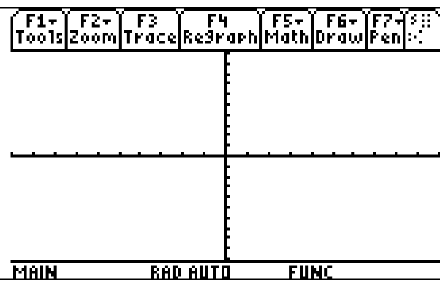
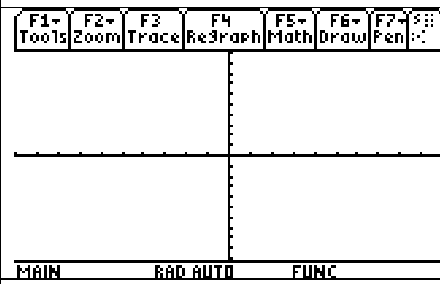
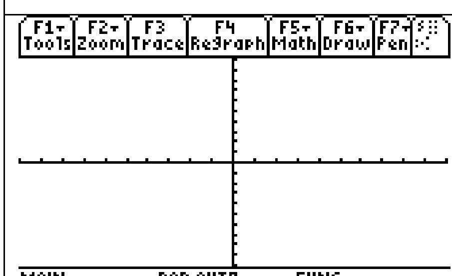
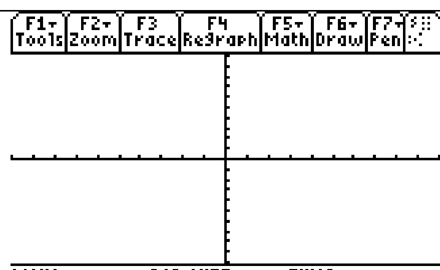
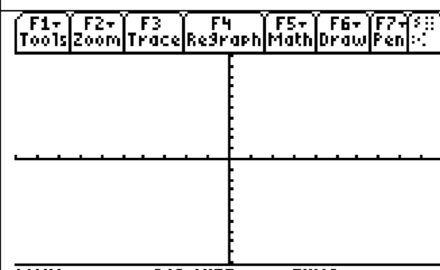
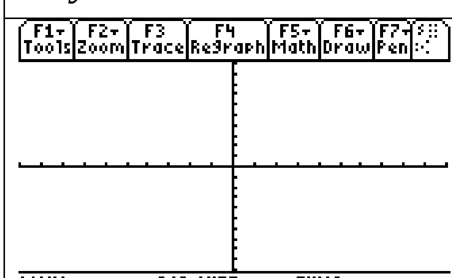
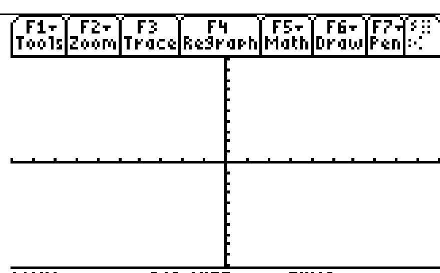
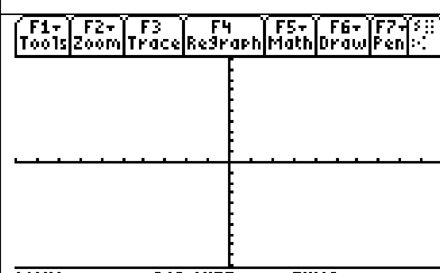


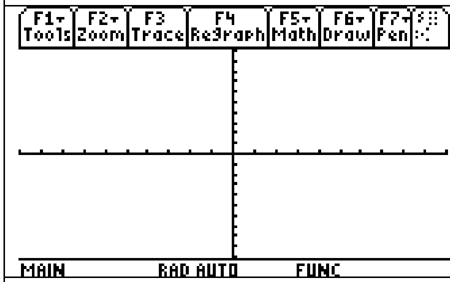
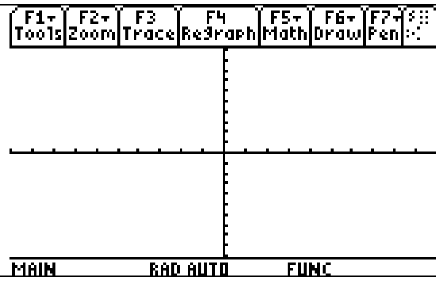
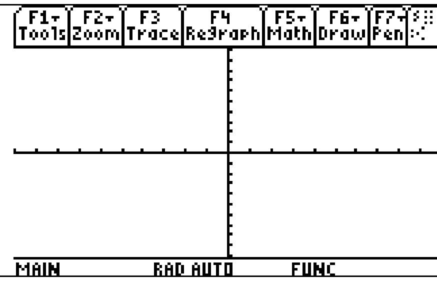
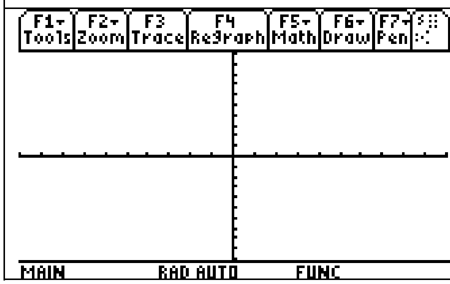
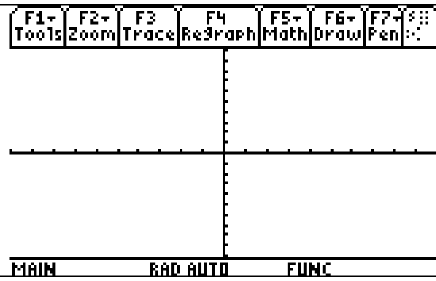
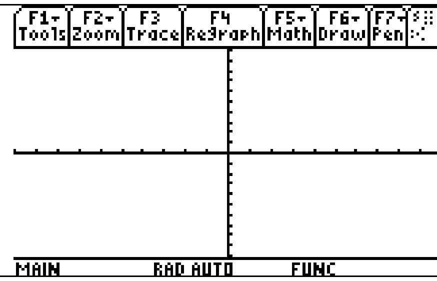


<u>TI 89 Derivative Lab (Karen Collins)</u>

<u>From the Home Screen</u>

Record the results on your lab sheet as you work. When you are finished taking the derivatives, graph each function and record the graph as accurately as possible. The given grid is ZoomStandard.

1. $y = x^2 - 4x + 4$	$y' =$	$y'' =$
		
2. $y = -3x^2 - x + 3$	$y' =$	$y'' =$
		
3. $y = 3x^3 + 8x^2$	$y' =$	$y'' =$
		
4. $y = -x^3 - 3x^2 + 1$	$y' =$	$y'' =$
		

5. $y = 0.25x^4 - 2x^2$	$y' =$	$y'' =$
		
6. $y = -x^4 + x^2 - 3$	$y' =$	$y'' =$
		

QUESTIONS:

1) If the graph of the original function is a parabola, then the first derivative graph is ________.

2) If the graph of the second derivative is a line with negative slope, then the original function's graph was ______.

3) If the graph of the original function is a parabola that opens up, then the first derivative graph is a line with ______ slope.

4) If the second derivative equation is a quadratic, then the original function must have been an equation whose degree was _____.

5) What would be true about a parabola whose second derivative graph was the line y = -3?

6) In general, to take the derivative of an expression raised to a power, you would . .

7) Knowing only the rule you wrote for question 6, what is the first derivative of $y = \sqrt{x}$?

<u>Rewrite each of the following so that you can use the power rule to find the derivative</u>.

8) Find the first derivative of $y = \sqrt[3]{x^5}$.

9) Find the first derivative of $y = \dfrac{2}{x^3}$.

Applications for Area under a Curve (Karen Collins)

Recall the problem: Example 4 p. 126
A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec.
It reaches a height of $s(t) = -16t^2 + 160t$ feet after t sec.

How high is the rock after 2 seconds? _____
This could be worded: **What is the total distance traveled after 2 seconds?**
In other problems we have completed a table:

t	***s***
0	
2	

TOTAL Distance: _____

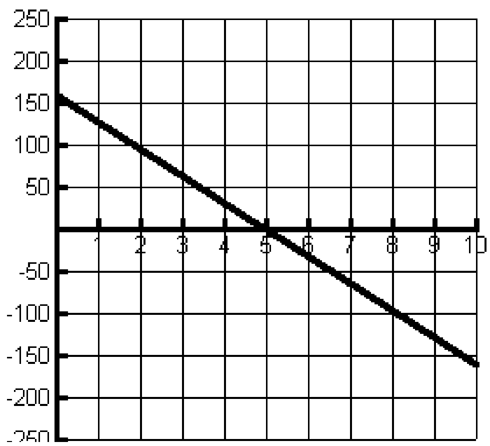
How high is the rock after 6 seconds? _____
Is it the same as: **What is the total distance traveled after 6 seconds?**

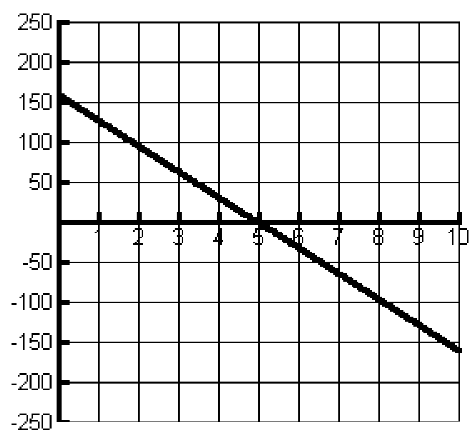
t	***s***
0	
6	

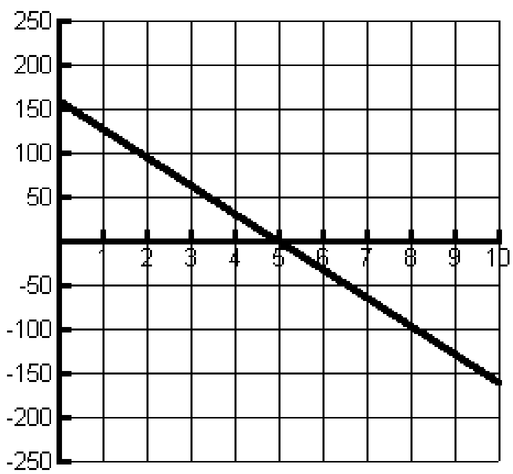
What additional point is important to note when doing total distance (besides the endpoints)?
TOTAL Distance: _____

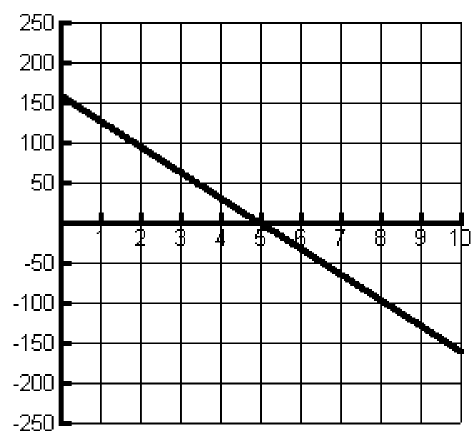
Examine velocity: ***v*(*t*) = -32*t* + 160**

Find the area under the velocity curve at $t = 2$, $t = 5$, $t = 6$, and $t = 10$.









Items of Interest

For TI 83-84 software emulator - Try http://wabbit.codeplex.com/ looks nice, works well, free. Not as good as TI-smartview which cost > $100
Lin Mc Mullin, National Math and Science Initiative, Dallas, TX www.nationalmathandscience.org

You are all invited to the **AP Calculus Reception and Panel Discussion at the NCTM** Annual Meeting.
Date: Thursday April 14, 2011
Time: from 6:00 – 8:00 PM
Location: Indianapolis Marriott Downtown, Marriott Ballroom 9/10
There is no charge and the meeting is open to all calculus teachers. The event is once again sponsored by D&S Marketing Systems. The format will be the same as in past years:
6:00 – 6:20 PM Reception with snacks and soft drinks
6:20 – 7:20 PM Panel discussion
7:20 – 7:50 PM Q&A
7:50 – 8:00 PM Door Prizes & Raffle
Confirmed panel members: Michael Boardman, Chief Reader for Calculus AB and BC, Lien Diaz, from the College Board, Kathleen Goto, member of the exam committee, table leader, Tara Smith, member of the exam committee, question leader. Come and meet the panel and ask them your questions. Lin McMullin will be the host.

http://courses.ncssu.edu/math/POW/POWindex.htm
NCAAMPT Calculus Challenge *Dan Teague*

Just making sure you all know about the **Park City Math Institute** for secondary teachers that will be held this summer. Information about the program an be found at mathforum.org/pcmi/hstp/ Applications are at pcmi.ias.edu/application-sstp/ It is a great way to spend three weeks, learn math and think about teaching. And if you are accepted, everything is paid! *Gail Burrill*

Approximately 3500 students from 27 states and 5 countries participated in an **ONLINE AP Calculus Competition** in October and November. Top 100 students and top 10 teams are posted at the following website: http://winplot-files-volume-of-revolution.wikispaces.com/Match+1+Results Match 1 is completed, but Match 2 is just about to start, (January 31st –February 4th). Approximately 1800 have signed up so far. For more information, teachers can contact me at pross@gcbe.org By the way, the competition is free! I expect next year the competition will start in the first week of November. I will begin the registration process next year in late September. If teachers will participate in the AP Calculus List Serve, then they will get an email automatically as the date of the competition approaches. Teachers who did not get a chance to participate can still have their students take the exam which is automatically scored by following the links and logging in the given information. *Paul Ross*

This year the **6th Mathematical Sciences Institute** (http://msi-no.org) will be held at Tulane University in a week-long format (July 24-29). Feel free to share this information with colleagues and contacts who might be interested in attending. This event is organized by metro New Orleans area teachers. Courses Offered: 1. Teaching Mathematics with a Tablet PC Leaders: Alyce Hesse and Nils Ahbel 2. Geogebra for Beginners Leader: Doug Kuhlmann 3. Visualizing Linear Equations Leader: Bruce Baguley 4. Activities and Alternative Assessments for Algebra II, Precalculus and Calculus Leader: Karen Bryant 5. NetLogo for Beginners Leader: Andy Talmadge 6. Physics for Math Teachers Leader: Ira Nirenberg 7. An Alternative to Precalculus Leader: Nils Ahbel 8. Geogebra - How to Use it in Precalculus and Calculus Leader: Doug Kuhlmann 9. Activities and Alternative Assessments for Algebra I, Geometry and Algebra II Leader: Karen Bryant 10. Revitalize Your Teaching with a Tablet PC Leader: Alyce Hesse

Wiki for AP Calculus Teachers (developed by Lin Mc Mullin). The Wiki contains videos on various topics in the AB and BC course and is intended for students (but teachers are welcome as well). The URL is http://apcalculusnmsi.wikispaces.com/ . Please take a look and if you like it give the URL to your students. *Lin Mc Mullin,* National Math and Science Initiative, Dallas, Texas.

An **Essay by Dan Kennedy** (a former ap-calculus test development chair:
http://www.math.uncc.edu/~hbreiter/kennedy.html
http://mail.baylorschool.org/~dkennedy/assessment
This was published in the Vol. 92 No. 6 (September 1999) issue of Mathematics Teacher.

WEBSITES

http://www.scribd.com/groups/fiew/9070-ap-calc-teachers
games *Susan M. Ferguson*

http://sss.houstonact.org
Volumes and Riemann sums *Dixie Ross*

http://clem.mscd.edu/
http://tinyur1.com/yga563
Solutions to FRQs *Lou Talman*

http://en.sikipedia.irh/siki/Ant_on_a_rubber_rope
Harmonic series *Seth Berg*

http://www.math.hmc.edu/funfacts/ffiles/10002,3shtml
Harmonic series *Paul Zorn*

http://keep2.sjfc.edu/faculty/lschlosser/ap_xummer_institute.html
http://www.manhattan.edu/programs/ap/
Summer institutes *Mark Howell*

http://www.thinkwell.com/a/calculusin20minutes
Free video

http://sss.calculusapplets.com/ *Richard Kilburn*

http://www.mastermathmentor.com
Free AP Calculus materials *Stu Schwartz*

http://education.ti.com/educationportal/sites/US/notProductSingle/syllabus_support,html
Ti website for Nspire

http://interactivemaths.net

http://wwwscribd.com/doc/3050815/Relating-function-and-1st-derivative
Card game *Susan M. Ferguson*

Bill Compton Workshops (http://home.montgomerybell.com/~comptob/mba/)
http://www3.davidson.edu/cms/x28009.xml
Calculus AB June 20-24, Davidson College, Davidson, NC:
http://agc.unca.edu/advanced-placement-institute
Calculus AB June 27-30, Un of North Carolina-Asheville, Asheville, NC:
http://www2.moreheadstate.edu/oce/index.aspx?id=2908
Calculus AB July 11-15, Morehead St University, Morehead, KY:
https://www.sdhc.k12.fl.us/apsi/
Calculus AB New July 25-29, Un of South Florida, Tampa, Fla:
http://www.drew.edu/cue/professional-certificates/ap-teacher-institute
Calculus AB August 1-5, Drew University, Madison, NJ:

http://www.stjacademy.org/page.cfm?p=278
St. Johnsbury, VT. I will be teaching a BC session July 17-22. AB sessions will be taught July 10-15, July 24-29 and July 31-Aug. 5. *Vic Levine*

http://iflipr.com/
The iphone/ipod app (iFlipr) used to be free, but now it's $4.99. I haven't tried quizlet - it might be better - it has better ratings on the iphone store. *Larry Rose*

http://msi-no.org
6th Mathematical Sciences Institute *Andy Talmadge*, director@msi-no.org or atalmadge1@gmail.com

Advanced Placement Program* Summer Institute - Sacramento State College of Continuing Education
Manuela McClary (916) 278-5485

Course related websites: http://apcentral.collegeboard.com/calculusab
http://apcentral.collegeboard.com/calculusbc
To search the list archives for previous posts go to http://lyris.collegeboard.com/read/?forum=ap-calculus

Attend the FIFTH U.S. Conference on CAS in Secondary Mathematics Come explore the future of mathematics education!

- Discover how secondary and middle school teachers are using CAS in their own classrooms.
- Get classroom tested ideas developed for CAS-enhanced classrooms.
- Learn what other countries are doing with CAS.
- Interact with prominent CAS pioneers from the USA and beyond.

WHEN: Saturday, June 28, 2008 8:15 AM - 4:15 PM
Sunday, June 29, 2008 8:00 AM - 1:00 PM

WHERE: New Trier High School (Northfield Campus)
7 Happ Road
Northfield, IL 60093
(approximately 20 miles from O'Hare Airport)

COST: Registration: $195 (before May 16, 2008)
$250 (after May 16, 2008)
(Fee includes continental breakfast, box lunch, snack, and conference shirt)

Optional Saturday evening Chicago-style dinner and Second City performance - transportation is included: $45

HOTEL: Renaissance Chicago North Shore Hotel, Northbrook IL.
$119/night conference rate available until June 5, 2008 (rate available for reservations June 26 to June 29, 2008)

Book directly online at

http://cwp.marriott.com/chinb/meecas/ or

http://cwp.marriott.com/chinb/usacas/

1-800-468-3571 Mention group code "USACAS" or "MEECAS"

HOW: On-line registration, updates, and hotel information available at http://meecas.org

For more information or questions, contact:

Ilene Hamilton at ihamilton@district125.k12.il.us