

The North Carolina Association Of Advanced Placement* Mathematics Teachers Newsletter

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Web Address

www.ncaapmt.org/calculus

ERROR – One of our Summer 2008 articles indicated that a course following BC Calculus was in the works – this is inaccurate as no course is presently planned by College Board – we regret this misstatement and apologize for any inconvenience this may have caused.

Notes from the President's Desk

I hope your year is going well. This is the time of year when “panic” begins to set in because time is at a premium. The following are suggestions that might make life a little easier for you and your students.

- An AP problem (from old exams) a day keeps the course content at the front of their minds.
- Have the students make study posters with important “stuff” for the classroom walls.
- Have the students make individual study cards (or posters for home.)
- Have the students make models for volume.
- Have students make sticky notes with important “stuff.”
- Go home each afternoon, and do something you really enjoy doing.

My wish is for each of your students to do her/his best as all prepare and take the exam.

*Gloria Nan Dupree, President
C. D. Owen High School, Black Mountain, NC 28711*

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Notes From The Secretary's Desk

Presently, I am still doing AP workshops and searching for full time work. One superintendent told me I was overqualified to teach high school (I guess because of my doctorate work). Another school system decided to hire a new teacher rather than me. (It was suggested they were looking for someone younger.) I am glad that there are places out there who seek out the best teachers – which may mean hiring someone who is a little independent and outspoken! I was excited to be accepted into the Consultant Mentor/Mentee program. So, my workshops should be getting some real improvements that I hope will benefit the teachers. If you know of any contract work or a job near my house where I do not have to travel much overnight – please let me know! I am doing free high school tutoring on Tuesday nights from 6-8 at my church with any donations going to missions – so far I have one student who is home-schooled. If you have items of interest, pass them on to me.

Deb Britt, Mars Hill, NC, dgb531@aol.com

Please remember to renew your membership to receive the two yearly newsletters. You can send your \$5.00 check, payable to NCA² PMT, to Jeff Lucia, 718 Lansdowne Road, Charlotte, NC 28270. Email address is jeff.lucia@providenceday.org.

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Meet the New Chief Reader, Mike Boardman Ben Klein, Davidson College, Davidson, NC

The 2008 Reading arrived with two significant changes, a new location and a new Chief Reader. For those of you who do not already know, the 2008 Reading was held in Kansas City, which will also be the site for the 2009 Reading. The new Chief Reader is Mike Boardman, who served as Chief Reader Designate for the 2007 Reading in Louisville and who served as Exam Leader for five years at Colorado State University.

Mike's undergraduate degree (in mathematics, of course) is from Western Washington University. He earned his Ph.D. at the University of Oregon under Bruce Barnes. He started his career on the faculty at Lake Forest College and moved to his current institution in 1995.

Mike is now Professor of Mathematics at Pacific University in Forest Grove, Oregon, which is a city of about 22,000, located about twenty-five miles from Portland. Mike's leadership and administrative skills are clearly just as obvious to his colleagues at Pacific University and the mathematical community in the Pacific Northwest as they are to the Advanced Placement readers, table leaders, question leaders, etc. with whom he has worked over the years.

At Pacific University, Mike has served as Department Chair, Chair of the Faculty of Arts and Sciences, President of the Pacific University Chapter of both the American Association of University Professors and also Sigma Xi. He currently serves as Chair of the Division of Natural Sciences. He served as Chair of the Pacific Northwest Section of the Mathematical Association of America and as President of the Oregon Mathematics Education Council.

As if all of the above, along with his work with the Advanced Placement Program, was not enough, Mike is serving or has served as an editor, referee, school board member, and the list of service and notable

accomplishments goes on and on. For example, Mike was recently a member of a team that provided professional development opportunities for K-12 teachers in the state of Oregon. Some other familiar names associated with this project are Tom Dick, a former Chair of the Test Development Committee, and Mike Shaughnessy, President-Elect of NCTM.

In light of the list above, one might wonder whether Mike had any time or energy left for teaching, but he obviously does. Mike's teaching was recognized with a University Award for Outstanding Teaching in 2007. His students, like his colleagues, also appreciate his dedication and hard work.

The paragraphs above make it clear that Mike is a high-energy person, and anyone who has seen him in action would agree with that characterization. He also has a wonderful sense of humor and takes great photographs. Not surprisingly, Mike's favorite activities involve motion. He enjoys running, hiking and kayaking.

At Mike's Pacific University Web page, one learns that "When not doing, learning, teaching mathematics, (Mike) spend(s) time with (his) family, Tami and Abby." This quote is accompanied by a picture of the two women in his life, taken, presumably by Mike, in front of the palace at Versailles.

Daughter Abby actually took the Advanced Placement Examination in 2008. If there is anything to the theory of genetics, we can be certain that she did very well on the examination. Abby is currently a first year student at the University of San Diego.

Mike and his wife, Tami, celebrated their twentieth anniversary last summer. One of their dreams is to travel by kayak from Washington to Alaska. The smart money says they will realize this dream.

Those of us who are involved with the Advanced Placement Program are grateful to Mike for his good work in the past and are looking forward to working with him in the years to come.

Things of Interest

2008 AP Calculus Exam Will be released – but not in time for 2009 exam. It could be 2010 before this is available. The 2008 Essays are available.

College Board released its 5th Annual AP Report to the Nation, which found that educators across the U.S. are continuing to enable a wider and more ethnically diverse population of students to achieve success in AP. The 5th Annual AP Report to the Nation uses a combination of state, national, and AP Program data to provide each state with the context it can use to celebrate its successes, understand its unique challenges, and set meaningful, data-driven goals to prepare more students for success in college. This year's Report includes participation and performance results in each specific AP subject for the class of 2008. The report is accessible online at www.collegeboard.com/apreport.

Handley High School Website (Winchester, VA) has a number of fun ways for kids to practice various calculus concepts. The web site is <http://www.doe.virginia.gov/Div/Winchester/jhhs/math/mathhome.html>

Volume Project

A scoring rubric is included with the project as well. I do usually allow 2 weeks to work on it.

http://mathisfun.org/volumes_of_solids_of_revolution%202006.htm *Sonya Land, Chesterfield, MO*

For an interesting derivative problem, consider the following Calculus Challenge Problem: <http://courses.ncssm.edu/math/POW/POWindex.htm> Find the derivative of $F(x) = \frac{\arctan(\sqrt{x}) + \sqrt{k}}{1 - \sqrt{kx}}$. In what way does the parameter k affect the value of the derivative? Graph the function for several values of k . *Dan Teague, NC School of Science and Mathematics*

AP Exams – Technology Issues

There is an article about calculator enhancements allowed and not allowed on the Statistics exam on AP Central: http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/197166.html The policy is not the same on the AP Calculus exam.

Non-graphing scientific calculators, computers, devices with a QWERTY keyboard, electronic writing pad, or pen-input/stylus-driven devices are not permitted for use on the AP Calculus Exams. Test administrators are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. Students should be thoroughly familiar with the operation of the calculators they plan to use on the exam. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the 2007-08 List of Graphing Calculators. Calculator memories will not be cleared. Students are allowed to bring to the exam calculators containing whatever programs they want.

WebAssign Comments

WebAssign is a web-based online learning management system which basically creates and grades assignments related to a certain textbook. The questions are randomized, the numbers are changed, and data is collected on student performance. The cost is \$8.80/student. You must be using the textbook to which you tie your assignments. www.webassign.net WebAssign can be used for its Discussion Board and for tests, quizzes and homework assignments. WebAssign can be used with FKDW book. The Larson 8th Edition has ready-made questions. You can write your own questions. You can see exactly what the student entered as an answer to a question. It computes all the grades and maintains a grade book. I've never encountered a problem getting on the system and I've been using it for three years. *Susan Gilleran*

Princeton Women in Mathematics Summer Program

From all received information, this is a new program this summer. <http://www.math.princeton.edu/swim/>

Logarithmic Differentiation Discussion: Dave V. writes that the modeling guy on his faculty threw an article about hat derivatives, (i.e., the logarithmic derivative) in his mailbox. The hat derivative is the correct quantity to look at when considering percentage growth and can be studied at the pre-calculus level, various places in calculus, and even in multivariable calculus. The article is in *The College Mathematics Journal*, Vol 33, No.1, pp. 32-37 by Stephen Maurer.

There is a discussion of the function defined by $f(x) = x^{1/x}$ in Arthur Knoebel's article "Exponentials Reiterated," *American Mathematical Monthly*, vol 88, no. 4, April 1981, pages 235-252. Finding the extrema of this function, and thus the ability to differentiate it, is important. The function is mathematically interesting because it is the inverse of the iterated exponential: if $x = y^{y^{y^{\dots}}}$ then $y = x^{1/x}$. Knoebel points out that this function also has applications to biochemistry. *David M. Bressoud*

A chapter on Graphing is posted on <http://www.jackmathsolutions.com/> Look in the "Free December" chapter section. This website offers many chapters, resources, etc. for AP Calculus as well as other topics which may be useful in high school math courses. Several chapters of the AP Calculus text I have written are available on the website. Feel free to check out the site and to contact me if you need any further information. I have taught AP Calculus at Havergal College in Toronto for the past decade. I am currently acting as a math consultant at Ryerson University, helping with transitions to university math. *Jack Koenka, Toronto*

See http://apcentral.collegeboard.com/apc/public/repository/2007_Annual_Participation.pdf for tables and data exhibiting AP exponential growth.

See <http://collegesearch.collegeboard.com/apcreditpolicy/index.jsp> for a table that summarizes the policies of Universities and Colleges regarding AP.

NBPTS shows a total of 2437 teachers certified in AYA Mathematics as of 12/04/2007 out of about 65,000 certified through the end of 2007. The recent press release by NBPTS.org said that nearly 15% of those teachers who hold national board certification teach either science or math.

For an application of computer technology used to solve problems using simulation – see the Dec.08/Jan.09 Mathematics Teacher on Pg. 390. It uses Fathom Dynamic Data Software from Key Curriculum Press.
Dave Kapolka

In an article in the March 2002 College Mathematics Journal, "Mixed Partial Derivatives and Fubini's Theorem," Asuman Aksoy and Mario Martelli refer to the two forms of the fundamental theorem as the Fundamental Theorem of Integral Calculus (the form that deals with differentiation) and the Fundamental Formula of Integral Calculus (the form that deals with evaluation). Martelli told me that this is common terminology in Italy.
Dan Kemp, South Dakota State University

Video to help with integrating on calculator:

http://www.associatedcontent.com/video/60155/evaluating_integrals_with_the_ti83.html?cat=4

Class assignments and notes for FDWK 2003: <http://www.ccsdk12.org/mclemens/> *Meg Clemens*

PowerPoint Lectures can be found at the website <http://online.math.uh.edu/HoustonACT/> and also at www.geocities.com/gkellymath
Greg Kelly, Hanford High School, Richland, WA

Sean Bird Shares His Work

Sean has some cool stuff and is a great presenter – he shares it all! His calculus website, located at <http://covenantchristian.org/bird/Calculus.htm> contains links to the “Stuff You Must Know Cold,” PowerPoints that are helpful for review, activities, calculus songs, SMART Notebook files, and more. There is also a link to his TI-Nspire CAS for Calculus webpage. There will be training for calculus teachers wanting to become better equipped at using the TI-Nspire CAS in the classroom in Indianapolis this summer. See <http://covenantchristian.org/bird/SmartTIap.htm> for more details. (Other workshops will be held throughout the country. See <http://tinspire2009.com/> .) Links to interesting places – see the pdf http://covenantchristian.org/bird/Smart/Calc1/Sec8_5CrossSection_SweetRotation_10_12.pdf. http://covenantchristian.org/bird/Smart/Calc1/Sec8_5CrossSection_SweetRotation_10_12.zip also shows this as a SMART Notebook. Sean has been writing many of the calculus files for timath.com. These files were written for the TI-Nspire CAS, and then a similar TI-89 activity was also written. They include accompanying worksheets and several include multiple choice AP exam-like questions.

Volumes of interesting shapes:

Try calculus on the campsite. <http://bird-godlydominion.blogspot.com/2008/07/calculus-on-campsite.html>
Other good stuff: <http://covenantchristian.org/bird/Calculus.htm>

Sean Bird's Calculus session at NCTM:

Improving Instruction with Technology: SMART Board™ and TI-Nspire™ Computer Algebra System (CAS). Hear and see how using the SMART Board and TI-Nspire CAS technology rejuvenated the speaker's teaching of calculus. Experience a hands-on activity with the latest hand-held learning tool, the TI-Nspire CAS. Learn about resources and professional electronic groups dedicated to make the technology more productive, enjoyable, and enlightening. This session (#65) will be Thursday, April 23, 2009, from 8:30-10 am at the NCTM annual meeting in Washington, DC in room 147A.

Graphing inverses of functions: For example, in order to graph the inverse of $Y1 = X + e^{(X)}$: $Y2 = \text{solve}(Y1(Z) - X, Z, 1)$ where solve is from the catalog. ZOOM 4. Graphing of the inverse, Y2, is slow because of solving (for Z which is Y2) for each value of X. You can speed up by setting Xres to 2 or 3 in WINDOW. The advantage is that you can use CALC 7 to find integrals of f-inverse. Or, on the home screen, use MATH 8 or 9 to find slopes or numerical integral values.

Drawing Vertical Lines on TI Graphing Calculator

If you need just a line, you could use the Line function under Draw Menu. Input Line (X1,Y1,X2,Y2) to draw a line between two points.

The next way is by using the APP Inequalz. When you go to the Y= window, there is an X= option in the upper left of the screen.

Use the draw menu from the graphing window and choose vertical – type in 5 if you want $x = 5$.

Graphing in parametric mode is another option. Using parametric mode: Let $X(T) = 5$, and let $Y(T) = T$. This graphs a line SEGMENT using the values of T designated in the window settings. You can lengthen or shorten the line segment by changing the Tmin and Tmax settings.

You can fake draw a vertical line: Use $y = 999(x-c)$ and this gives the appearance of a vertical line. You can then shade inequalities as well with this.

Mathematics Holiday Carol

Olivia Judson writes a very good blog for the NY Times (usually on Biology). A recent one, however, was on Newton and featured the following:

On the tenth day of Newton,
My true love gave to me,
Ten drops of genius,
Nine silver co-oins,
Eight circling planets,
Seven shades of li-ight,
Six counterfeiters,
Cal-Cu-Lus!
Four telescopes,
Three Laws of Motion,
Two awful feuds,
And the discovery of gravity!

The entire piece is here: <http://judson.blogs.nytimes.com/2008/12/23/the-ten-days-of-newton/?8ty&emc=ty>

Lin Mc Mullin

Teaching in China: Want an adventure to add to your career experiences? If anyone is interested in teaching AP Calculus in China, please dash me an email and I will be happy to let you know how great it is here. I am at an international dream-school in a coastal area with actual blue skies, on a 2-yr contract.

leec@aian.org.cn

Changing the way we do Proofs? *Notices*, American Mathematical Society [AMS]. See the website <http://www.ams.org/ams/press/hales-nots-dec08.html> for a ground-breaking collection of four articles by leading experts, published in the *Notices* of the American Mathematical Society, that explore new developments in the use of formal proof in mathematics. The four *Notices* articles explore the current state of the art of formal proof and provide practical guidance for using computer proof assistants. If the use of these assistants becomes widespread, they could change deeply mathematics as it is currently practiced.

The four articles are:

Formal Proof, by Thomas Hales, University of Pittsburgh

Formal Proof---Theory and Practice, by John Harrison, Intel Corporation

Formal proof---The Four Colour Theorem, by Georges Gonthier, Microsoft Research, Cambridge, England
 Formal Proof---Getting Started, by Freek Wiedijk, Radboud University, Nijmegen, Netherlands

The articles appear in the December 2008 issue of the Notices and are freely available without a subscription. For Further Information Contact: Professor Thomas Hales, Department of Mathematics, University of Pittsburgh. hales@pitt.edu

Upcoming Conferences

NCTM Washington, DC April 22 - 25, 2009 www.nctm.org	NCCTM Greensboro, NC October 29 - 30, 2009 Speaker forms due April 15	AP Calculus Summer Institute UNC Charlotte July 6 - 10, 2009 David Royster - droyster@email.uncc.edu
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Applets for Visualizing Calculus Concepts Ellen Vesey, The Westminster Schools, Atlanta, GA

Topic	Link
Secant to Tangent	http://www.calvin.edu/~rpruim/courses/m161/F01/java/SecantTangent.shtml
Secant to Tangent	http://math.furman.edu/~dcs/java/tangent.html
Riemann Sum	http://www.cut-the-knot.org/Curriculum/Calculus/RiemannSums.shtml
Fundamental Theorem	http://www.math.dartmouth.edu/~klbooksite/4.04/404.html
Fundamental Theorem	http://ugrad.math.ubc.ca/coursedoc/math101/notes/integration/ftc.html
Average Value	http://www.calculusapplets.com/aveval.html
Volumes of revolutions (disks)	http://www.math.dartmouth.edu/~klbooksite/4.08/408.html
Volumes of Rotation	http://www.math.psu.edu/dlittle/java/calculus/volumedisks.html
3D Solids and Cross-sections	http://www.slu.edu/classes/maymk/banchoff/SolidsOfIntegration.html
Volume and Square cross-sections	http://www.ies.co.jp/math/java/samples/renshi.html
Volume and Cross-sections	http://www.runet.edu/~wyang/MAPLE/java/java/rhansan/rhansan.html
Length of a curve	http://xanadu.math.utah.edu/java/ApproxLength.html
Slope Field grapher	http://www.math.ust.hk/~amoy/math150/SlopeField.html
Slope field applet	http://www.batesville.k12.in.us/Physics/CalcNet/diff_eqn/euler_applet.htm
Euler's Method	http://www.calculusapplets.com/euler.html
Improper Integrals	http://www.calculusapplets.com/improper.html
Taylor Polynomials	http://www2.norwich.edu/frey/TaylorPolynomials/
Maclaurin Approximations	http://mathinsite.bmth.ac.uk/applet/macseries/Maclaurin%20Series.html
Infinite Series	http://webcompmath.sourceforge.net/wcm/calculus_applets/Series.html
Integral Test tutorial	http://www.jtaylor1142001.net/calcjat/Solutions/Series/IntTest/IntTest1/IntTest1Layers.htm
Integral Test	http://www.calculusapplets.com/integraltest.html
Lots of nice visual "mathlets"	http://www.math.umn.edu/~rogness/mathlets.shtml
More Links	http://www.ima.umn.edu/~arnold/graphics.html

NCAAPMT Calculus Problems Testimonial

Ellen Vesey, The Westminster Schools, Atlanta, GA

I can HIGHLY recommend Dan Teague's Calculus Problems of the Bi-Week that are posted on the NCAAPMT website. These are great, non-traditional Calculus problems that are accessible to both AB and BC Calculus students. Dan runs a competition for classes whose teachers are members of NCAAPMT, but even if you are not a member, you can access the problems (and solutions) on the organization's website: <http://courses.ncssm.edu/math/POW/POWindex.htm>

I have used the problems in several different ways. Sometimes I assign them as independent projects for students to complete and submit for a grade. Other times I will break the class in to groups and either have each group work on the entire problem, or on occasion I have had the groups work on different parts of the problem, and in the end the students must come together and write a class solution. Some of the trickier questions we have worked together as a large group, with students racing up to the board to contribute ideas as they have their "ah-ha" moments.

Just to spark your interest a bit I have listed below a brief description of each problem we have done so far and the calculus involved. If any sound interesting to you, go to the website and check it out!

From 2007-2008	<i>Calculus Required</i>	<i>Brief Description</i>
Problem 1	NONE!	Students are asked to write the numbers 1 – 50 using 1, 9, 7, 9 in that order
Problem 2	NONE – Just IVT	Basketball free throw questions about making a given percentage of shots
Problem 3	Def of Derivative	Use the definition of derivative to develop definitions for the nth derivative
Problem 4	Geometric Series + Derivative Rules	A little intro to power series using geometric series and derivatives
Problem 5	Derivatives, Max/Min	An interesting look at the concept of the curvature of a function using derivatives and parametric equations
Problem 6	Derivatives, Min/Max	A favorite of my students! Look at how bees forage for pollen. Uses a piecewise function
Problem 7	Related Rates	Drop a brick from a building and follow it with your eyes....
Problem 8	Integrating Polynomials	How can we represent trig functions using polynomials (not using Taylor). Good review of trig identities and inverse trig functions
Problem 9	Area between curves	Areas as probability.
Problem 10	Parametric Derivatives	Another favorite! Explore a “Butterfly Curve” described parametrically.
Problem 11	Derivatives and integrals	Good AP review question that has kids work with calculus on a generically described function
Problem 12	Integration with Parametric Equations	Curvature and length of a curve using parametric equations.
Problem 13	Solving a differential equation	Using a boat and water skier application, solve a differential equation
Problem 14A	Differential Equations	A modeling question using differential equation – explore the battle of Trafalger
Problem 14B	Probability and Integration	Crossing a street using calculus!

From 2008-2009	Calculus Required	Brief Description
Problem 1	NONE	Basketball shooting scenario for working with percentages.
Problem 2	Definition of Derivative	Use the definition of derivative with a twist – think about using a “delta-y” rather than “delta-x”
Problem 3	Product Rule and Chain Rule	Students develop rules for nth derivative products.
Problem 4	Min/Max and Sigma Notation	Using derivatives to find min/max of functions defined as sums
Problem 5	Min/Max	A twist on the traditional “box” problem. Look at boxes that might not be square!
Problem 6	Limits and L’Hopital	A great problem that has the students look at probability density functions. I had my AP Stats teacher do a one class intro to the concept before we looked at this question
Problem 7	Inverse Trig derivatives and basics ideas of integration	A two part questions. First part has students do chain rule derivative with inverse tangent and explores the affect of a constant. Second part asks students some basics using integral notation and properties
Problem 8	Slope Fields and Differential Equations	A twist on the logistic function – the Gompertz Curve!

I don't look at the problems as something *extra* that I have to add to an already busy schedule of topics. These problems are great for helping students solidify their understanding of AP Calculus topics in a less traditional context. Give them a try!

Exploring Numerical Derivatives

Deborah Britt, Mars Hill, NC

(Adapted from H. Smith Risser article in October 2008 *Mathematics Teacher* – p. 224)

The following worksheet (and the article it came from) helps students explore calculator errors on the TI-84 when finding derivatives. It also explores when this happens and why. Solutions are provided.

Consider the following multiple-choice question:

Given $f(x) = |x|$, find $f'(0)$. (a) 0 (b) -1 (c) 1 (d) undefined

Ninety percent of students in a recent class selected choice (a). Rather than thinking about this geometrically, the majority of students selecting (a) indicated they used the numerical derivative feature on their TI-84 calculator.

Discussion Questions

When is it appropriate to use your calculator? How does your calculator compute this?
 When is the numerical derivative a good approximation for the slope of tangent line?
 What is the limit definition of a derivative? Why does that matter?

Try these. After using the calculator, it might be wise to sketch a few secant lines and use these lines to explain why the approximation is accurate (or not).

1. $f(x) = x^3 - 4x + 7$ at $x = 2$. $f'(2) =$ _____
Is the approximation reasonable? Why or why not?
2. $f(x) = 3\sqrt{x-4}$ at $x = 5$. $f'(5) =$ _____
3. $f(x) = \sqrt{x}$. $f'(0) =$ _____
4. $f(x) = |x - 3|$. $f'(3) =$ _____
5. $f(x) = \sqrt[3]{x-3}$. $f'(3) =$ _____
6. For the problems in which the numerical derivative is not an accurate approximation of the exact value of the derivative, what characteristic does each function have at the indicated point that causes difficulty in calculating an accurate numerical derivative?
7. Sketch a continuous function $p(x)$ with the following conditions:
 - $p'(0)$ is undefined
 - The symmetric difference $\frac{f(0+1) - f(0-1)}{2}$ exists
8. How might you determine whether the numerical approximation of the derivative is reasonable without finding the exact value of the derivative?

Possible Solutions:

1. 8.000 - approximation is close to exact value – so reasonable
2. 1.500 - reasonable
3. Error: Non-real answer. The exact value is undefined. However, when the symmetric difference $\frac{f(0+h) - f(0-h)}{2h}$ is calculated, $f(-h)$ is complex.
Disagreement as to whether answer is reasonable.
4. 0 but exact is undefined - so not reasonable
5. 100 but exact is undefined so not reasonable approximation
6. Problems 3-5 – exact value of derivative is undefined
7. Any function with a cusp or a vertical tangent line at $x = 0$ would fit the criteria – you can make this more interesting by adding in some additional criteria.
8. Answers vary – discuss possibilities.