The North Carolina Association Of Advanced Placement Mathematics Teachers Newsletter

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Notes from the President's Desk

I would like to thank those who presented at the NCAAPMT session of the NCCTM state conference in Greensboro in October 2006. I would also like to thank those who attended the session and to extend an invitation to all our members to join us at the 2007 conference.

Recently I was having a discussion with fellow alumnus, and of course we reminisced about our collegiate days, our professors and calculus. We both recalled our teachers lecturing and demonstrating problems that we were to recall and duplicate at some later time. We both remembered doing lots of symbolic manipulation and graphing by hand - but what beautiful graphs! We then compared the way we were taught and the way we now teach. In a typical AP Calculus class, we ask our students to solve familiar and unfamiliar problems, making connections to other mathematical ideas and to the real world; express solutions numerically, analytically, and graphically; explain their thinking and justify their reasoning. This sounds very similar to the NCTM process standards: Problem Solving, Reasoning and Proof, Communications, Connections and Representations. The AP curriculum and expectations have changed over the years; especially the level of thinking required of students and the communications of their mathematical understanding and reasoning. I think AP calculus teachers as well as the students face a very challenging but very rewarding experience.

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According to recent reports, our state and nation need more students having this experience in order to fill the future economic, political and social demands that require higher levels of mathematical understanding. That's great news for the AP program, but the question for most schools might be "where are the students?" If we are to increase the number of students in our AP calculus program, I believe we need to better prepare our current students in all our mathematic classes. We need to use the processes that we know will deepen their understanding of mathematics and allow them to continue their study of mathematics. We need all our students experiencing the same thinking and learning processes as employed in AP calculus, the same high level of expectations, and the same access to a rich curriculum. As AP calculus teachers, we must enlist the help of all our colleagues (including middle school teachers) in order to better prepare all our students. Working together in our schools, districts and professional organizations, we can make it happen.

Ray Jernigan, NCAAPMT President **jernigan@northcarolina.edu**

Notes from the Secretary's Desk

Many thanks to Trish Morris for helping me last summer with the newsletter. I am finishing the final course on my Ph.D. this semester and look forward to becoming a more normal person again. The stress of teaching, retiring, changing jobs and states, as well as taking doctoral courses for the past three years has been much more than I wanted. For those of you who know me well, normal does not really mean normal! My dissertation topic has been approved and I have a chair and two committee members so far. My comps committee is organized and ready for me to take the written part the first weeks of June with Orals to follow by the end of the month or early July. I have completed one mini-pilot study and am in the process of doing pilot 2. I am writing drafts this semester of chapters 1 and 2 of my dissertation but expect some major re-writes will be needed.

Anyway, I still live in North Carolina but am retired from NC after 26+ years. I am now teaching Calculus with two students at University School on the campus of East Tennessee State University. I also have 4 other classes. We are a K-12 school with 300 in grades 9-12. It is so wonderful to be at a place where they really treat you as a professional. We are small and everyone knows each other so well. No one stresses over end of course testing. They have an Algebra I Gateway test that everyone must pass to graduate – but no other math EOC's at the high school. I have a SmartBoard and 10 computers in my classroom. We have ordered the TI Navigator system. I feel like the teacher I was when I started many years ago – I actually decide what to teach, when to teach it, how to teach it and for how long. There is no extra money for National Board Certification (which I renewed last year) and the 10% from Asheville City School is gone as well. But, their pay is not all that bad. The scale does not show the 10% they contribute to retirement as part of your salary – they just put it away for you without ever showing it on your paycheck. Their health insurance is not free but I have mine with NC. They also have cheaper gas taxes so I often buy gas in TN. They have no state income tax so my paycheck is actually more per month take home than before, but as a NC resident, I will have to pay NC taxes on it. Anyway, teaching calculus is the same!

We are on a year round schedule, which everyone loves, but it is taking me some time to adjust – we start around July 10 and go nine weeks on and 3 weeks off, and we end in May. I loved the 3 weeks off in October and at Christmas. I am looking forward to the three weeks coming up in March – just in time for NCTM and T^3 conferences. They do not have as many workdays and they pay for subs when they need you to be in meetings. They are also generous with travel money to conferences compared to NC places I have been. The year round schedule is unique to our school – the only lab school in the state. I was late arriving (as my summer courses were not complete until August) so some students taking calculus switched out before I arrived. I am looking forward to building this program. I hope you enjoy the newsletter and that it will add some excitement to the upcoming review season for the 2007 AP exams! The winter newsletter is usually smaller than the summer one but we try to include things for you classroom. Please plan now to attend our sessions in the fall at NCCTM and share ideas with us. *Deborah Britt*

Send comments or articles to **dgb531@aol.com**

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Local Linearity and L'Hôpital's Rule

Daniel J. Teague - The NC School of Science and Mathematics, Durham, NC

The historical role of calculus as the lynchpin of the mathematical preparation of mathematicians, engineers, and scientists is being pitted against the emerging role of calculus in the general education of the informed citizen. For the calculus instructor, this creates a tension between the need to present arguments that are informative and engaging at the level of the student's understanding, and the desire to present arguments that are mathematically rigorous, but perhaps, not as illuminating. Finding effective ways to introduce the principles of calculus to students from diverse backgrounds and with a multitude of ambitions and employment goals is the challenge many reform calculus texts attempt to meet. The teacher of calculus and the writer of calculus textbooks must decide whether to present arguments that will convince the students they meet in their classes or the colleagues they meet in the hall.

Local Linearity

Students who have used graphing calculators extensively in their preparatory courses are familiar with the principle of local linearity. They know from much experience that if they "zoom in" on just about any section of one of the functions they have studied, the function quickly "becomes linear". Calculus adds to this experiential base by explicitly expressing the "zoom line" as the tangent to the curve at $x = a$,

$$y = f(a) + f'(a)(x - a).$$

If the function is indistinguishable from its tangent line in some small region around $x = a$, then the function is said to be locally linear. The principle of local linearity is closely related to that of differentiability; a function is locally linear at all points at which it is differentiable. By emphasizing the visual, geometric aspect of local linearity and the greatly simplified algebraic operations with linear functions, we can offer students intuitively appealing, convincing arguments as a basis for their understanding of calculus. Such an approach falls short of the criterion for rigor desired by many practicing mathematicians, but is in line with Ron Douglas's comment in *Toward a Lean and Lively Calculus* that "I believe that students should learn that the fundamental notion of the differential calculus is how effective linear and quadratic approximation is for studying nice functions and how this can be used to study systems that change."

To illustrate the approach consider the standard and local linearity approach to the weak form of L'Hôpital's rule:

L'Hôpital's Rule (Weak Form)

The traditional development follows this general path:

Theorem: Assume that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and $g'(a) \neq 0$. Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

Proof: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f(x) - f(a)}{g(x) - g(a)}$ since $f(a) = g(a) = 0$. So

$$\lim_{x\to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x\to a} \frac{\frac{f(x) - f(a)}{x-a}}{\frac{g(x) - g(a)}{x-a}} = \frac{\lim_{x\to a} \frac{f(x) - f(a)}{x-a}}{\lim_{x\to a} \frac{g(x) - g(a)}{x-a}} = \frac{f'(a)}{g'(a)}.$$

My students often commented after this development, "I see *that* it's true, but I don't see *why* it's true." What insight does this derivation give the student? Compare the formal proof above to the less formal argument below.

The simplest of all cases is $\lim_{x\to 0} \frac{ax}{bx}$, where both f and g are linear functions. Either by algebraic simplification or by the geometry of similar triangles, it is easy for students to see that the ratio of ax to bx is always the constant $\frac{a}{b}$ for all non-zero *x*. Consequently, the limit is just $\frac{a}{b}$.

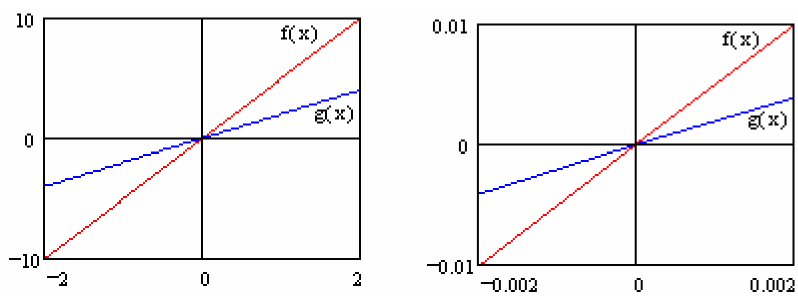


Figure 1: Example of $f(x) = 5x$ and $g(x) = 2x$ on $[-2, 2]$ and $[-0.002, 0.002]$

Now, consider $\lim_{x\to 0} \frac{f(x)}{g(x)}$ with arbitrary functions *f* and *g*. In this case the ratio of $f(x)$ to $g(x)$ is not constant, but changes with *x*. However, if *f* and *g* are differentiable at $x = 0$, then they are locally linear near $x = 0$. This means both functions behave as if they were linear functions near $x = 0$ and we know how to handle the ratio of linear functions. Since $f(x) \approx f(0) + f'(0)(x-0) = f'(0)x$ and $g(x) \approx g(0) + g'(0)(x-0) = g'(0)x$ near $x = 0$, we expect that $\frac{f(x)}{g(x)}$ behaves like $\frac{f'(0)x}{g'(0)x}$ and

$\lim_{x\to 0}\frac{f'(0)x}{g'(0)x}=\frac{f'(0)}{g'(0)}$. If you zoom in on f and g around zero, the geometry gets closer and closer to that of the simple linear case.

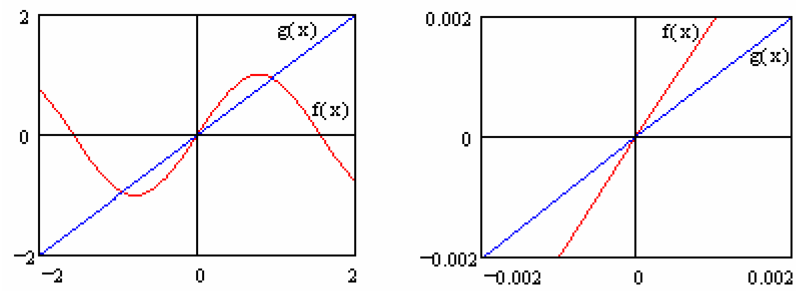


Figure 2: Example with $f(x)=\sin\left(\frac{x}{2}\right)$ and $g(x)=x$ on $[-2,2]$ and on $[-0.002,0.002]$

If we move away from the origin to $x=a$ with $f(a)=g(a)=0$, then we have

$$\lim_{x\to a}\frac{f(x)}{g(x)}\approx\lim_{x\to a}\frac{f(a)+f'(a)(x-a)}{g(a)+g'(a)(x-a)}=\frac{f'(a)}{g'(a)}.$$

With this argument, students can "see" why L'Hôpital's Rule is true. Simply put, if you zoom in on differentiable functions, they "behave" locally as if they are linear. As a general approach to a question in calculus, we reduce the problem to its linear approximation, and ask the question of this simple form. In doing this, students can have a tool to assist their investigation into many aspects of differentiable calculus.

There is more to it than this, of course. Henry Pollak once commented that, "In introductory calculus, you cannot tell the whole truth. If you could tell the whole truth, there wouldn't be a course called Analysis." Given that we can't tell the whole truth in an introductory calculus course, each of us must decide what part of the truth to tell. Our effort has been to present the part that gives insight and understanding; the part that offers compelling arguments that make the fundamental truths of calculus believable and understandable, and which offer a basis on which to build future work.

Conclusion

The linearity approach takes advantage of and builds upon the geometric intuition developed by using graphing tools in preparatory courses. Entering students are comfortable with the both the principle of local linearity and the algebraic manipulations of linear functions. By considering the behavior of the tangent lines, students can go a long way towards determining the behavior of non-linear functions and the rules that govern differential calculus. If greater rigor is desired, this approach is a very useful way to investigate the problem. Students can reduce the problem to the linear approximation, "follow the lines", and make a conjecture about how well the behavior of the linear model represents the behavior of the function. They now have a conjecture which requires more rigorous justification.

References

Bartkovich, Kevin, et al, *Contemporary Calculus Through Applications*. Providence, Rhode Island: Janson Publications, 1995.

Douglas, R. G., "Opening Remarks at the Conference/Workshop on Calculus Instruction, in Douglas, R. G. (ed), *Toward a Lean and Lively Calculus*, (MAA Notes Number 6). Washington, DC: Mathematical Association of America, 1987.

The Two Cone Problem
Lin McMullin - Peekskill, NY

"Find the maximum sum of the volumes of two cones that you make out of a construction paper disc with an area of S."

Suggested by Dr. Jing, St. Anthony High School, Long Beach, CA, in AP Calculus EDG 11/12/06

Lin McMullin's Solution

You can download and view at the website: www.LinMcMullin.net
http://www.linmcmullin.net/PDF_Files/Two_Cone_Problem.pdf.

Assumption: The cones are formed by cutting a sector from a circular disk and forming both pieces into cones. Another approach which may involve some waste may be possible.

For the moment assume S = 1

Then the radius of the disk is $r=\sqrt{\frac{1}{\pi}}$ and this becomes the slant height of both cones.

The circumference of the disk is $2\pi\sqrt{\frac{1}{\pi}}=2\sqrt{\pi}$

Let x = the arc length of the sector cut from the disk. $0\le x\le 2\sqrt{\pi}$, or about $0\le x\le 3.5449$

Cone 1:

Slant height = $\sqrt{\frac{1}{\pi}}$

Circumference = x

Radius of base $r_1=\dfrac{x}{2\pi}$

Height = $\sqrt{\left(\sqrt{\dfrac{1}{\pi}}\right)^2-\left(\dfrac{x}{2\pi}\right)^2}=\sqrt{\dfrac{1}{\pi}-\left(\dfrac{x}{2\pi}\right)^2}$

Volume is $V_1=\frac{\pi}{3}\left(\frac{x}{2\pi}\right)^2\sqrt{\frac{1}{\pi}-\left(\frac{x}{2\pi}\right)^2}=\dfrac{x^2}{12\pi}\sqrt{\dfrac{1}{\pi}-\left(\dfrac{x}{2\pi}\right)^2}$

Cone 2:

Slant height = $\sqrt{\frac{1}{\pi}}$

Circumference = $2\sqrt{\pi}-x$

Radius of base $r_2=\dfrac{2\sqrt{\pi}-x}{2\pi}$

Height = $\sqrt{\left(\sqrt{\frac{1}{\pi}}\right)^2-\left(\frac{2\sqrt{\pi}-x}{2\pi}\right)^2}$

Volume is $V_2=\frac{\pi}{3}\left(\frac{2\sqrt{\pi}-x}{2\pi}\right)^2\sqrt{\left(\frac{1}{\pi}\right)-\left(\frac{2\sqrt{\pi}-x}{2\pi}\right)^2}=\dfrac{\left(2\sqrt{\pi}-x\right)^2}{12\pi}\sqrt{\left(\dfrac{1}{\pi}\right)-\left(\dfrac{2\sqrt{\pi}-x}{2\pi}\right)^2}$

The quantity to be maximized is $V=V_1+V_2=\dfrac{x^2}{12\pi}\sqrt{\dfrac{1}{\pi}-\left(\dfrac{x}{2\pi}\right)^2}+\dfrac{\left(2\sqrt{\pi}-x\right)^2}{12\pi}\sqrt{\left(\dfrac{1}{\pi}\right)-\left(\dfrac{2\sqrt{\pi}-x}{2\pi}\right)^2}$

Then (computation done with TI-Interactive)

$$\frac{x^2}{12\cdot\pi}\cdot\sqrt{\frac{1}{\pi}-\left(\frac{x}{2\cdot\pi}\right)^2}+\frac{\left(2\cdot\sqrt{\pi}-x\right)^2}{12\cdot\pi}\cdot\sqrt{\frac{1}{\pi}-\left(\frac{2\cdot\sqrt{\pi}-x}{2\cdot\pi}\right)^2}$$

$$\frac{x^2\cdot\sqrt{4\cdot\pi-x^2}}{24\cdot\pi^2}+\frac{\left(x-2\cdot\sqrt{\pi}\right)^2\cdot\sqrt{-x\cdot\left(x-4\cdot\sqrt{\pi}\right)}}{24\cdot\pi^2}$$

$$\frac{d}{dx}ans$$

$$\frac{x\cdot\sqrt{4\cdot\pi-x^2}}{12\cdot\pi^2}-\frac{x^3}{24\cdot\pi^2\cdot\sqrt{4\cdot\pi-x^2}}+\frac{\left(x-2\cdot\sqrt{\pi}\right)\cdot\sqrt{-x\cdot\left(x-4\cdot\sqrt{\pi}\right)}}{12\cdot\pi^2}-\frac{\left(x-2\cdot\sqrt{\pi}\right)^3}{24\cdot\pi^2\cdot\sqrt{-x\cdot\left(x-4\cdot\sqrt{\pi}\right)}}$$

$$\text{solve}(ans=0,x)$$

$$x=2.39631 \text{ or } x=1.77245 \text{ or } x=1.1486$$

$$\left(\frac{x^2\cdot\sqrt{4\cdot\pi-x^2}}{24\cdot\pi^2}+\frac{\left(x-2\cdot\sqrt{\pi}\right)^2\cdot\sqrt{-x\cdot\left(x-4\cdot\sqrt{\pi}\right)}}{24\cdot\pi^2}\right)\Big| x=\{2.39631, 1.77245, 1.1486\}$$

$$\{.082007, .081434, .082007\}$$

These are the volumes at the three critical points (end point volumes = 0, i.e. no cones). The first and third are the same because they represent the solution when cone 1 and cone 2 are interchanged by cutting so that $x = 2\sqrt{\pi} - 1.1486 = 2.39631$

Finally, since the area was multiplied by a factor of $\frac{1}{S}$ to make it equal to1, we must multiply the volumes by a factor of $S^{\frac{3}{2}}$ t o return to the original problem size. **The maximum volume is** $(0.082007)S^{\frac{3}{2}}$.

Dr. Jing's solution:

I wrote the problem with two purposes: 1) to write a single valuable function to represent the volume of two cones made out of a unit disc, to find the maximum and to compare the difference between computing by hand and computing by a graphing calculator; 2) to magnify or reduce from a unit disc to a disc with area S.

Let r_1 and r_2 be the radii of the two cones made out of a disc with an area S. Then we have $r_1 + r_2 = R$, where R is the radius of the disc.

$$V = f(R, r_1, r_2) = \frac{\pi}{3}\left[r_1^2\sqrt{R^2 - r_1^2} + r_2^2\sqrt{R^2 - r_2^2}\right]$$

For a unit disc, $R = 1 \Rightarrow r_2 = 1 - r_1$. Then $V = g(r_1) = \frac{\pi}{3}\left[r_1^2\sqrt{1 - r_1^2} + (1 - r_1)^2\sqrt{1 - (1 - r_1)^2}\right]$

With a graphing calculator (TI-83 plus), I got $V = V_{max}$ at $r_1 = 0.3240$ or $1 - 0.3240$.

Therefore $v_{max} = g(0.3240)$.

For a disc with area S, $V*_{max} = f\left(R, 0.3240R, (1-0.3240)R\right) = f\left(\sqrt{\frac{S}{\pi}}, 0.3240\sqrt{\frac{S}{\pi}}, (1-0.3240\sqrt{\frac{S}{\pi}}\right) = h\left(S\right)$

Therefore, $V*_{max}$ is the function of S only.

First, I would like to tell you that I read your solution carefully and found that it leads to the same results as my approach would do. The only difference between the two approaches is that you used S = 1, while I used R = 1. The reason I used R = 1 is that I found this way it is easier to simplify the total volume expression, and that the expression is more readable.

Second, I want to say that I like your final formula for the maximum volumes. It is neat! I should have thought about the way of using similarity.

Third, I want to make it clear that the cut I made is exactly the same as yours. Therefore, there is no waste.

Now let me show why your approach is the same as my approach:

The circumference of the disc is $2\pi R$ and the circumferences of the two cones are $2\pi r_1$ and $2\pi r_2$.

Since the two sectors are cut from the center of the disc, we have $2\pi R = 2\pi r_1 + 2\pi r_2$, which leads to $R = r_1 + r_2$. Therefore, r_1 and r_2 are exactly the radii of the two cones.

By the way, the slant heights of the two cones in my approach are the same as the radius of each disc, as shown here:

$V = f\left(R, r_1, r_2\right) = \frac{\pi}{3}\left[r_1^2\sqrt{R^2 - r_1^2} + r_2^2\sqrt{R^2 - r_2^2}\right]$, and $H_1 = \sqrt{R^2 - r_1^2}$ and $H_2 = \sqrt{R^2 - r_2^2}$.

Solution by Ken Sterling using the central angle:

Wow! My first reaction to the problem was that it would be a messy confirmation that the max would occur using half the circle for each cone. I wrote the sum of the volumes as a function the central angle, t, of a circular sector cut for one of the cones.

$V\left(t\right) = \text{Constant} \cdot \left[t^2\sqrt{4\pi^2 - t^2} + \left(2\pi - t\right)^2\sqrt{4\pi t - t^2}\right]$. $V\left(t\right) = 0$ at $t = 0$ and $t = 2\pi$ as expected, but that is where my expectations for the graph ended.

Using technology, the graph appears to have *three* horizontal tangents. One is at $t = \pi$, but that looks to be a relative min. Two relative max's occur. One is at approximately t = 2.03584 and the other at t = 4.24753. The graph of the derivative (again using TI-89) confirms three horizontal tangent lines at those locations.

I did not try to solve the derivative set to zero by hand, but I found the symbolic derivative with the TI-89 and asked it to solve deriv = zero. It just went busy for about ten minutes and never produced an answer. The apparent answer is so unintuitive that I have to wonder about mistakes in my methods or something else going on. Wow again! Nice problem.

Solution by Mark Snyder:

The only thing I would add to this discussion is that the function to be minimized can be made to look nicer by taking $\frac{1}{2} - x$ to be the fraction of the original disk cut out to make cone #1, so $\frac{1}{2} + x$ is the fraction cut out to make cone #2. This means that the sum of the volumes of the cones must be symmetrical under

$x \to -x$, i.e., it must be an even function of x. Then when the cones are formed, the volume to be minimized is proportional to $F = (2x-1)^2 \sqrt{4-(2x-1)^2} + (2x+1)^2 \sqrt{4-(2x+1)^2}$.

This makes the function to be minimized look nicer, and it becomes more apparent that it's even. It also leads to an exact solution.

From a graph of the function, one sees that it has three local extrema. Owing to the above symmetry, this means that one of the extrema must be at x = 0, while the other two are at opposite values of x. The extremum at x = 0 is a local minimum, corresponding to cutting the disk in half (the solution I thought I would find as the maximum...), while the other two are maxima, and occur at approximately $x = \pm 0.176$, i.e., make the cut at an angle of about 118.6°. That works out to a maximum volume of about $0.08 \cdot S^{\frac{3}{2}}$, as others have found. In fact, the problem can be solved analytically: with some manipulations, if we set $F = 0$ and square to remove the square roots, we find that $y = 4x^2$ obeys the cubic equation $9y^3 - 87y^2 + 51y - 5 = 0$. As a cubic equation, this has an analytic, exact solution (which I will *not* quote...). While this is a cute problem, I was hoping that there was some "aha" method of finding the solution, not a graphical/numerical/analytical one. I looked for such a solution, but couldn't find one.

Calculus questions that are mainly for CAS machines?

I use 89s in my class and want to really push thinking beyond just some button pushing. I use old AP Problems and some of the things from Calculus for a New Century, but would appreciate knowing if there is a dedicated "CAS resource." I know that the forthcoming UCSMP textbooks for Advanced Algebra, FST (Functions, Statistics, and Trigonometry), and Pre-Calculus all integrate CAS and assume that students have access to CAS at home and in the classroom. But I haven't found any perfect resource yet for Calculus.

USA CAS conference in June will probably have more ideas. http://meecas.org/
The following links may be of interest:
http://www.connecting-t3.com.au/t3_activitylist.php?psearch=A:3
http://www.mbuescher.com/professional/
Sean Bird, Covenant Christian High School
seanbird@covenantchristian.org
http://cs3.covenantchristian.org/bird

There are some I've been collecting at http://www.linmcmullin.net/ Click on "Resources" and then "CAS" and then on "Problems for CAS solution."
Lin McMullin

AP Calculus Blog

Readers are invited to "look over my shoulder" as I teach Calculus. I share my weekly assignments, my reasons for the classroom choices I make, etc. I talk about how and why I do what I do, and describe my successes, my failures, and my frustrations. The blog is sponsored by Peoples Education. You can find the latest blog and links to earlier columns at http://www.peoplescollegeprep.com/blog/index.php.
Alan Lipp, Easthampton, MA

SmartBoards

Here are two websites for SmartBoard:
http://www.amblesideprimary.com/ambleweb/yahtzee/index.html
http://its.leesummit.k12.mo.us/smartboard.htm
Karen Sandrock, Morganza, MD

I've been using one for a while...there's so much you can do with it...it's awesome, especially with programs like Winplot, TI-Smartview, and Calculus in Motion (must-haves for having smartboard). You can take a look at my website for school at the address below for some ideas.
http://cmsweb1.loudoun.k12.va.us/51520826123424280/site/default.asp?

I don't put up my calc stuff since I don't use it in the same way with the kids since I have a small class. Another good resource is Tom Reardon...look at his website at http://www.austintown.k12.oh.us/~aust_tr/
Dan Muscarella, Leesburg, VA

The AP Audit

I found that there are 4 things they are looking for:

1. Sufficient AP content. A one or two page syllabus linked to your text should work. Chances are you already have that done.

2. Multiple Representations. Be sure to use the words Graphically, Numerically, Analytically, and Verbally. (also be sure to implement the ideas in your course!)

3. Communication. You must show evidence that you require your students to "explain solutions both verbally and in written sentences."

4. Calculators. Include something to show that you teach the four ways AP expects students to use their calculators.

Along with a syllabus showing that you cover the topics of AP calculus (item 1 above), you should describing your approach to items 2, 3, and 4. Then include copies of a few activities to reinforce your claims regarding calculators, communication and multiple representations. Your school administrator must set up an account for the school and give each AP teacher the school code. Then you must download and print a form that both you and the principal sign. One part of it says that you have read the course description. Another part to be signed says that the AP students all have access at school and at home to graphing calculators. This is a good chance to obtain some calculators if your school has not been supporting you here. The signed form must be faxed. When you log in to the audit site you will see - yes or no - whether your signed form has been submitted. There is also a column for your syllabus that says one of the following: unsubmitted, being transferred (means sent to reviewer), under review, approved. Mine was approved so I am not sure what it says if not approved. You do get a chance to resubmit. Email me if you have questions. brittd@etsu.edu

Unit Circle

I've created a Flash animation to assist students in learning the unit circle. You can find it on my website at http://www.dudefree.com/unitcircle/. Note that you need the most current version of Flash Player to use it (there is a link on that page to download the current version). My email address is dude@dudefree.com.

Information on Slope Fields

Most of my sources are somewhat vague on that topic.
http://apcentral.collegeboard.com/apc/public/teachers/development/44676.html scroll down, it's near the bottom of the page.
http://apcentral.collegeboard.com/apc/public/homepage/11871.html
http://apcentral.collegeboard.com/apc/public/repository/slopefieldhandoutapnc11870.pdf
If you want a slopefield program for a TI-83+ or TI-84, there are several available at ticalc.org that various teachers and students have written
http://www.ticalc.org/archives/files/fileinfo/388/38861.html

Free Materials

All of the activities in the books from the link below are free for downloading. The first book is the one I have had the most compliments on: Calculus Activities with the TI-84.
http://education.ti.com/educationportal/sites/US/sectionHome/activitybook_section_calculus.html

Website: www.houstonact.org Included there are talk materials from previous workshops by Dan Kennedy, Peter Horton and Jeff Morgan. Check it out!

Calculus In Motion update reminder

If you are a Calculus In Motion user, and you haven't yet requested your free 2006 Free Response animations, you can do so by emailing Audrey Weeks at amweeks@aol.com. Be sure to include your 5-digit serial number from the front of your Calculus In Motion CD-ROM and use an email account that accepts attachments (that's how you'll receive the update). Visit the calculusinmotion.com website for more information and look for the next updates to Calculus In Motion and Algebra In Motion in spring of 2007.

Audrey Weeks
Calculus In Motion & Algebra In Motion
Burbank, CA
amweeks@aol.com
www.calculusinmotion.com

MAA Wokshops

Every year the Mathematical Association of America (MAA) runs a series of Professional Enhancement Programs (PREP workshops) of interest to faculty members. The list of workshops for the coming summer is available at http://www.maa.org/prep/2007/ Which workshops will be of interest will of course depend on what other courses the AP teachers teach, the characteristics of the individual schools they are in, and how they run their courses. Titles that may be of interest are:

Mathematics from Asia's Past
Flash at the Beach: Creating Mathlets with Adobe Flash
The Geometry of Vector Calculus
The Genius of Euler
Exploring Multivariable Calculus Using Maple
Calculus: Online and Interactive

Mike May, S.J., Saint Louis University

Free Progams

I have FREE programs and several other things at:
http://www.mvhs.fuhsd.org/jon_stark/calculus/TIprograms/catalog.htm
All but one of these are programs that I wrote myself (one is by a student), and I'm happy to share them with other teachers for no charge. I've tried the programs in the FDWK resources, but they are very crude and not very user friendly, so I cooked up these instead. The LRAM, MRAM, RRAM program has a pretty good user interface, and gives you the option of graphing the fit of the rectangles to the curve (it adjusts the window to fit the situation, too). The slope field program is rather crude (written by one of my students), but all the others (Newton's method, Simpson's rule, etc.) are pretty well developed with good error-catching built-in and good interfaces for easy use, and largely self-explanatory. There are counterparts for the TI-89 on that webpage, too. These are set up to download as .zip files, which you can de-zip and transfer to your calculator with TI-connect or any equivalent.

Jon Stark

iPods in Calculus

I ran across a student the other day who had taken the DVD (of notes, lectures) etc. that came with his calculus textbook (Larson ?), loaded it into his iPod and was using that to listen to the DVD and study his calculus.

Lin McMullin
Peekskill, NY
www.LinMcMullin.net

Identities Matching Activity

An identities cut-up matching activity, along with some other great activities for Trig and Calculus, can be found at http://www.pen.k12.va.us/Div/Winchester/jhhs/math/lessons/mtrig.html

LaDonne Harris

A New Resource for AP Calculus Teachers and Students

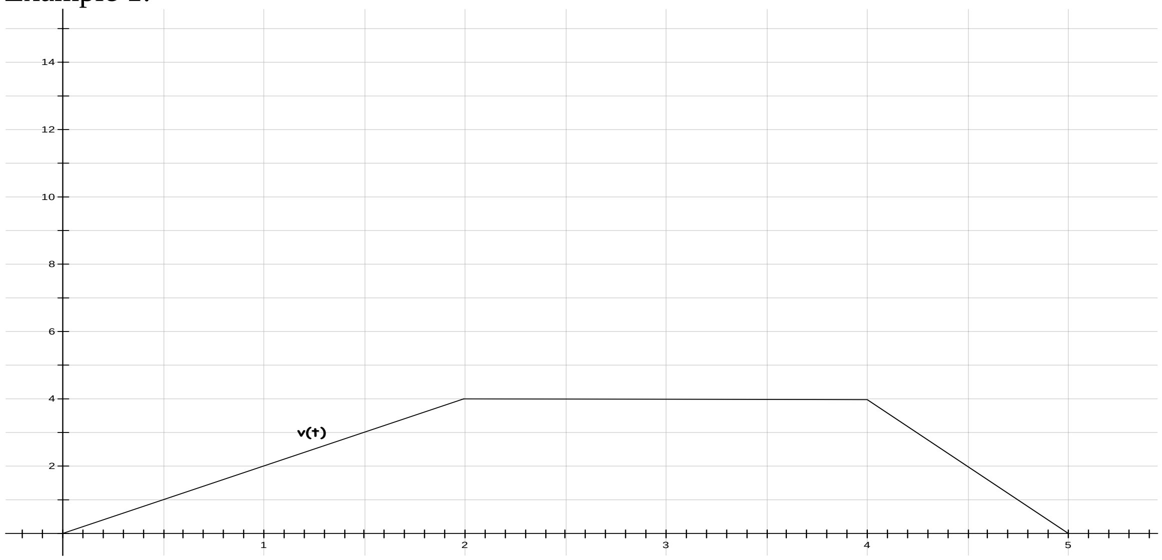
Be Prepared for the AP Calculus Exam, an AP Calculus review book, has been available since 2005 and received great reviews. The authors are two veteran AP Calculus teachers, Mark Howell and Martha Montgomery, who served on the AP Calculus Exam Development Committee. (Mark also wrote the latest version of the *AP Calculus Teacher's Guide*.) The book has nine review chapters that cover all AB and BC topics, as well as five practice exams: three AB and two BC. The practice exam contributors (Benita Albert, Thomas Dick, and Joe Milliet) are also veteran teachers and former Development Committee members. A review of *Be Prepared* is online at AP Central, and there are several reviews at amazon.com. *Be Prepared* is published by Skylight Publishing and is available through their web site, www.skylit.com, as well as amazon.com and other booksellers. Chapter 10, "Annotated Solutions to Past Free-Response Questions," is available for free at www.skylit.com/calculus.

AB Calculus Worksheets

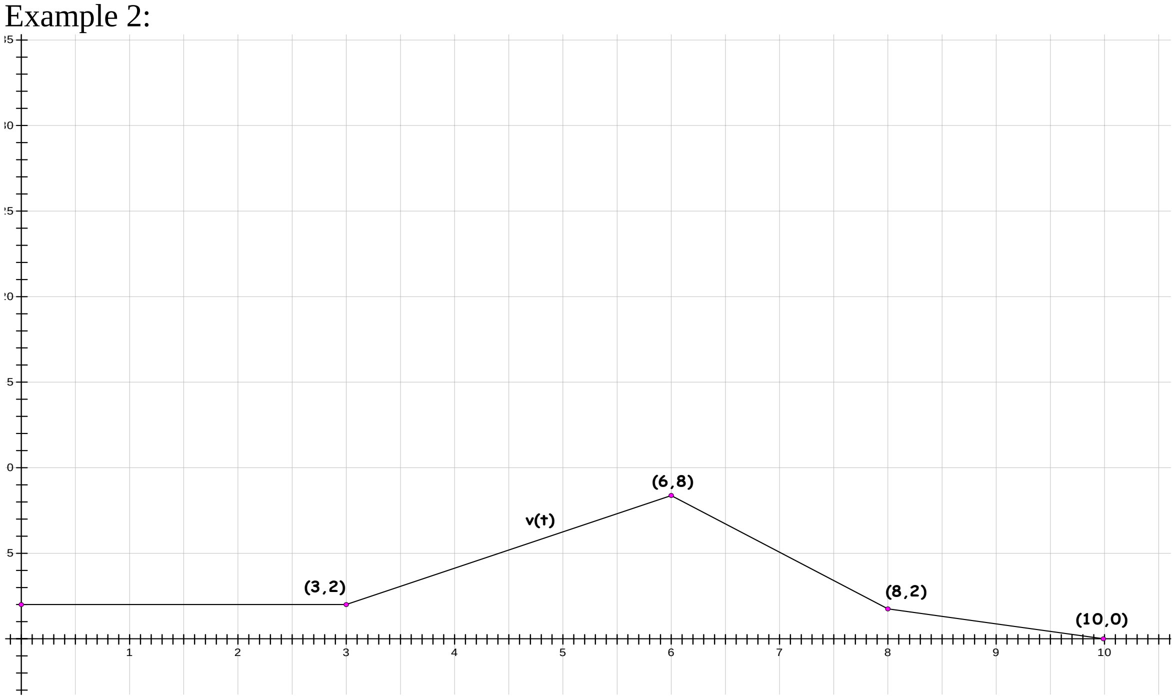
What Motivates the Definite Integral???

At the end of each line segment, place a point on the grid that would identify the distance traveled on that interval. Use formulas from geometry to help you. Continue to accumulate the distance as you move from left to right. Assume your starting position, or initial condition is $s(0) = 0$.

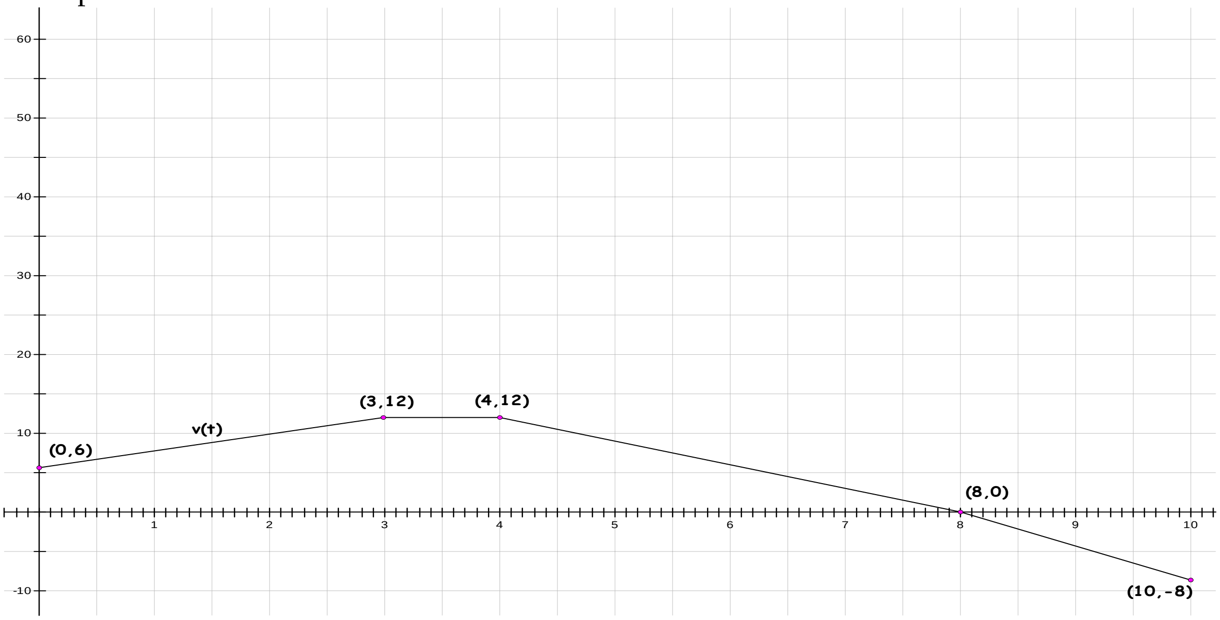
Example 1:



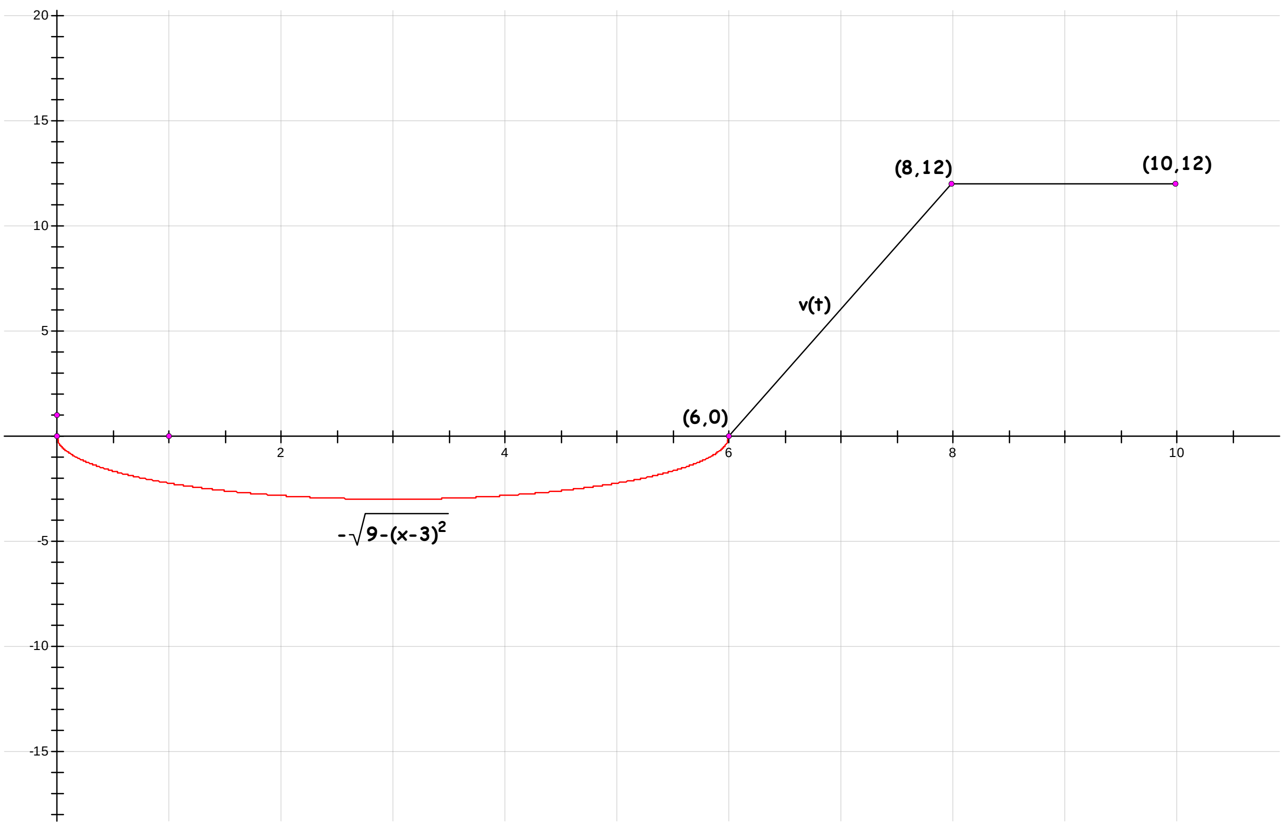
Example 2:



Example 3:



Example 4:



CALCULUS SUDOKU #1

	8							Clue #1
	Clue #2	Clue #8	8		9			1
		1	Clue #11	$\int_2^3 2x\,dx$			Clue #3	
8	1	6	7			$\sum_{k=-2}^{2}(2k+1)$		
9					1		Clue #4	
			Clue #9	Clue #5				
	Clue #6				8		Clue #12	Good # to approx. π or e
Clue #7							8	
			3	1			Clue #10	6

Clue #1: If $f(g(x)) = \left(2x^3 - x\right)^4$, find $g'(1)$

Clue #2: $\lim_{h\to 0}\frac{(x+h)^2 - 4}{h} = ?$

Clue #3: Find $f'(4)$ if $f(x) = \frac{x-5}{x-3}$.

Clue #4: What is the y intercept of the line tangent to $y = 4\sqrt{x} + \frac{8}{\sqrt{x}} - 3$ at $x = 4$?

Clue #5: Find $f'(3)$ if $f(x) = (2x-7)^3$

Clue #6: Find the slope of the line normal to the curve $\left(x^2+4\right)y = 8$ at (2,1). (This curve is called the Witch of Agnesi.)

Clue #7: If $f(x) = (x-1)^{2/3}$, then $f(x)$ is not differentiable at $x = ?$

Clue #8: If $f(x) = 8\sin^6 x\cos x - \cos^2 x$, find $f'\left(\frac{\pi}{4}\right)$

Clue #9: If $f(x) = \begin{cases} ax^2 + b \\ -4x \end{cases}$ is continuous and differentiable at x = -2, what is $a + b$?

Clue #10: $\lim_{x\to -\infty}\frac{\sqrt{16x^2 + x - 1}}{3 - 4x} = ?$

Clue #11: The y-intercept of the line tangent to $f(x) = \frac{x}{x-1}$ at (2,2)

Clue #12: Find $f'(1)$ if $f(x) = x^3 + x^2$