The North Carolina Association Of Advanced Placement* Mathematics Teachers Newsletter

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Remembering Charlie Bodine

Jeff Lucia, Providence Day School, Charlotte, NC

The news came to me suddenly, tersely, in an e-mail message from a colleague: Charlie Bodine had passed away in Summerville, South Carolina on February 10, 2006, after suffering a stroke a few days earlier. Perhaps many of you do not know who Charlie Bodine was, but those who have been members for many years will remember that Charlie was the founder and inaugural president of NCA^{2}PMT.

I remember receiving a letter from Charlie in the winter of 1992. He introduced himself as a calculus teacher at Charlotte Country Day School. He was interested in starting a statewide network of AP Calculus teachers similar to one he had belonged to when he taught in New England. I told him I would be interested, and later that year a group of eight calculus teachers from Asheville to Wilmington got together and established the NCA^{2}PMT. Charlie was quickly elected president, eventually serving on the Board for six years. Through Charlie's leadership in those early years our group grew in numbers, became recognizable as an affiliate of NCCTM, and our semi-annual newsletter became a staple for both experienced and novice calculus teachers in North Carolina and across the United States.

I recall sharing rides with Charlie to and from several Board meetings. Charlie always talked about different ways he could help his students learn, and he felt very strongly about professional development for teachers. His vision to create the NCA^{2}PMT grew from this. If at any time during your membership in NCA^{2}PMT you feel you have benefited from some aspect of the organization, please pause for a moment and pay tribute to the man who got it all started.

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Notes From The Secretary's Desk

Deb Britt, Asheville High School, Asheville, NC

Well, it has been a busy year for me so far. My dissertation topic and prospectus are coming along and I am getting excited about finally getting to my research part of doctoral work. I still have this last summer to take 3 classes at University of Louisville. I am looking forward to having Dr. Bill Bush who does lots of research with assessment. Also, I will finally get to meet Alan DeYoung, whose books inspired my dissertation topic. He will be my rural research teacher. I am exempt from the Calculus course next fall so that leaves me with only 2 classes, comprehensive exams and dissertation. Anyone interested in ACCLAIM (Appalachian Consortium Center for Learning, Assessment and Instruction in Mathematics) doctoral program, they are starting Cohort 3 in the summer of 2007. I am not sure where all the funding will be coming from with the NSF cutbacks for this new cohort. But I have enjoyed having all my expenses paid except for ½ of my tuition. I have not enjoyed all the time it takes and feeling like NCAAPMT is taking a back seat. I look forward to getting back to our Board meetings soon. Asheville High is going well, with too many kids in my AP classes (over 30). My principal cancelled our BC section so the year had a rough start for me. Grading 30 AP papers has been way too time-consuming. I hope you find some interesting information in this newsletter. Please write and let me know if you have suggestions. Please share with other teachers – especially the new ones. deborah.britt@asheville.k12.nc.us

Letter from the President

Trish Morris, Greensboro Day School, Greensboro, NC

Thanks to those of you who were able to join us at the NCCTM state meeting in Greensboro in October. We hope you find those presentations helpful and we welcome any of your thoughts about how to make them more beneficial for you.

About a year ago I happened to read an article about math anxiety on the mathacademy website. It was entitled "Coping With Math Anxiety". As the title implies, it was written for the student. As a math teacher, I jotted notes, highlighted, and found myself identifying with nearly everything that was written. I was impressed and relieved that someone was able to put into words my observations of thirty years of teaching math as well as my own fears and feelings of being a student of math for fifty years. (Heaven forbid that my students may find out that I share their feelings – what a fraud!) In the article, the author defined math anxiety and related some of its social and educational roots. He mentioned that no one would admit to being illiterate but a vast majority "can't do math". Some of the common myths and misconceptions are described as:

1) The aptitude for math is inborn. To a small extent this is true; many people choose their life's work because they are good at it and enjoy doing it.
2) To be good at math you have to be good at calculating. With today's technology, that argument doesn't hold water.
3) Math requires logic, not creativity. How did all this science, used to describe and quantify other sciences, known as math come into being???

4) In math, what's important is getting the right answer. How many times have you written on papers "Show your work."???
5) Men are naturally better than women at mathematical thinking. Is this another bias that needs to be unlearned? In an opinion piece in one of the weekly news magazines a female engineering student wrote about finding out that some of her male classmates actually work harder than she did, but did so late at night and alone so they wouldn't be "found out".

The author then describes steps to be taken to address math anxiety and strategies for studying math. http://www.mathacademy.com/pr/minitext/anxiety/ is the website where you can find this article. It is well worth the time it takes to read it!

By now, I'm sure your schools have received the information about the AP Course Audit that will start to take effect next year. College Board has also determined that it will not be offering dual administration of AP exams in 2007 and 2008.

Please remember to renew your membership to receive the two newsletters. A form can be found in this newsletter. You can send your $5.00 check, payable to NCA ^{2} PMT, to Jeff Lucia, 718 Landsdowne Road, Charlotte, NC 28270. His email address is jeff.lucia@providenceday.org.

Good luck in finishing your curriculums and heading towards review time. And good luck to your students on the exam. We always welcome your comments and look forward to helping you in any way.

Sincerely, Trish tmorris@greensboroday.org

Lesson Plan for Computing π
Brad Huff, University High School, Fresno, CA

A lesson plan that has worked for me uses the Monte Carlo technique to find the value of π. Before random number generators were available on graphing calculators, I used phone books to generate random numbers - just as the physicists at Los Alamos did (the Manhattan directory) when this technique was invented to determine interaction times and distances for neutrons in fissionable material.

Each student in class does the following steps:
Step One - construct a unit square on a piece of graph paper about 5 inches by 5 inches; i.e., each half inch is 0.1 units.
Step Two - obtain a random number between 0 and 1. This is the x-coordinate. Using a phone book, you select a page at random, select a number at random, and use the final four digits as 0.xxxx. Check your calculator handbook on how to obtain a random number between 0 and 1.
Step Three - obtain a random number between 0 and 1. This is the y-coordinate.
Step Four - plot the point.
Step Five - square the x-coordinate number and square the y-coordinate number; add them together.
Step Six - if the sum of the squares is less than or equal to one, it is "in". If it is greater than one, it is "out".
Step Seven - Repeat this process nine more times.
Step Eight - the teacher obtains the results from each student, listing the number of "in"'s and "out"'s in two columns.

What are we doing? If one were to throw darts at a square target with a quarter circle of unit radius drawn with the center are one corner, the ratio of the number of darts "in" the circle to the total number thrown would be in the ratio of the area of the quarter circle to the area of the square. Since the area of the quarter circle is $\pi/4$ and the area of the square is one, the value of π should equal 4 times the ratio of the total number of "in" points to the total number of points generated by the class. In a class of 20 to 40 students, the value usually comes out surprising close to 3.14. I would encourage you to try this with your classes.

With programming skills, either on a computer or using the capabilities of their calculators, students can write a program that does this, draws the square and quarter circle, and plots the random points generated.

An added benefit of doing this is that "random" number generators usually generate pairs of points that, when plotted, form a pattern of bands across the square. If the random number generator were truly random, the points generated should be random and not form a pattern of bands. The Monte Carlo technique was also used in the past to find areas under curves using the same idea of generating random points, using the function value to determine if the random point was "below" or "above" the curve over the range of values of x of interest, and expressing the area as the ratios of points to the ratios of areas.

Nuclear Waste Disposal (Euler's Method Problem)

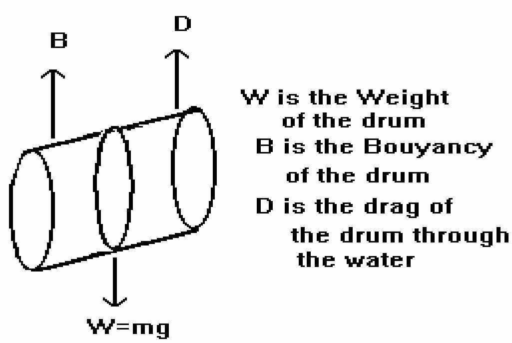
teague@ncssm.edu
Dan Teague, NC School of Science and Mathematics, Durham, NC

For many years, the United States "solved" its nuclear waste disposal problem by filling 55 gallon drums with the waste and dropping them off in the Atlantic Ocean at a depth of 300 feet. When questioned about the practice, the Atomic Energy Commission (now the Nuclear Regulatory Commission or NRC) argued that the drums would never leak from prolonged exposure to the sea. While satisfied about the deterioration of the drums, a number of engineers questioned whether the drums would withstand the impact when they land on the bottom of the ocean. Tests were done, and the engineers found that if the barrel landed with a velocity greater than 40 feet per second, the drums were susceptible to rupture. How can we determine the velocity of a barrel after it has fallen through salt water to a depth of 300 feet? (Note: this is an authentic problem, so units of measure are those used at the time.)

I use this problem as our first example of the power of Euler's Method to investigate real-world problems we don't yet know how to solve analytically. Later in the course, we revisit the problem and compare the solutions approximate and analytic solutions.

Defining the Mathematical Problem

What do we know about this situation? We always know that $F = ma$. Further, we know that $a = \frac{dv}{dt}$. So we always have the differential equation $\frac{dv}{dt} = \frac{F}{m}$ at our disposal. We will use this equation often in calculus. In each physical situation, the forces are different, so we need to determine specific expressions for *F* and *m*. What are the forces acting on the drum as it falls?



The force causing the drum to descend is due to gravity, and is measured by the *weight* of the drum. The weight of a 55 gallon drum filled with nuclear waste is approximately 527 pounds. Two forces act to retard the descent. The first is the *buoyancy* of the drum. Buoyancy is the force of the water acting on the drum. Its magnitude is the weight of the water displaced by the drum. A 55 gallon drum filled with sea water weighs approximately 470 pounds. The second force is the *drag* force of the water acting on the drum. The drag force depends upon the velocity of the object moving through the water, the faster it moves, the greater the resistance. By using towing experiments, the relationship between the drag force and velocity of the drum was determined to be $D = .08v$. The orientation of the drum as it fell did not appear to affect this relationship. Naturally, other objects moving through other mediums will have a different functional representation for drag.

In this development, we have chosen down as the positive direction. Consequently, as the barrel falls, its velocity increases.

Developing the Model

Students typically view $F = ma$ as just an equation from physics. We need for them to see it as a differential equation. Since $F = ma$, we know that $\frac{dv}{dt} = \frac{F}{m}$. This statement is always true. What changes from problem to problem is the representation of the force, *F*. In this specific case, we know that

$$F = W - B - D = (527 - 470 - 0.08\text{ v}),$$

and that $W = mg$ so $m = \frac{527}{32.2} \approx 16.366$. This gives the differential equation $\frac{dv}{dt} = \frac{32.2}{527} \cdot (57 - 0.08\text{ v})$.

From the structure of the differential equation, we can see that the terminal velocity is $v = \frac{57}{0.08} = 712.5$ ft/sec. So, it is clear that given a long enough fall, the barrel will clearly exceed 40 ft/sec and break open when it hits the bottom. The real question is: how fast is it going when it has fallen 300 feet?

Euler's Method Solution

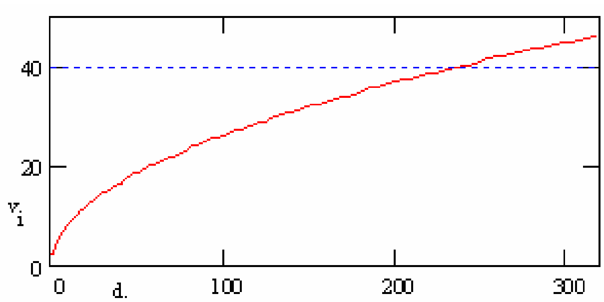
We cannot solve $\frac{dv}{dt} = f(v)$ for $v(t)$ early in the AP Calculus course, so we "solve" the approximate equation $\frac{\Delta v}{\Delta t} = f(v)$ by using Euler's method instead. If $\frac{\Delta v}{\Delta t} = f(v)$, then $\Delta v = f(v) \cdot \Delta t$. Remember, Δv is the change in velocity (the difference in successive *v*-values), so $\Delta v = v_n - v_{n-1}$. This generates the equation $v_n - v_{n-1} = f(v_{n-1}) \cdot \Delta t$. This is finally rewritten as Euler's formula $v_n = v_{n-1} + f(v_{n-1}) \cdot \Delta t$.

This iterative equation generates velocities as a function of *time*. We want to know the velocity as a function of *distance*, specifically, what is *v* when *d* is 300! How do we get from velocity to distance. What is the relationship between velocity and distance?

Since velocity is the derivative of distance with respect to time, we know that $\frac{d}{dt}d = v$. This is another differential equation. We solve the approximate equation $\frac{\Delta d}{\Delta t} = v$ as before.

$$\Delta d = v \cdot \Delta t \quad \rightarrow \quad d_n - d_{n-1} = v_{n-1} \cdot \Delta t \quad \rightarrow \quad d_n = d_{n-1} + v_{n-1} \cdot \Delta t.$$

This iterative equation uses the values generated previously for velocity to generate depths. We now graph *v* against *d* (using $\Delta t = 0.1$), and see how fast the drum is moving at 300 feet.



$t_{133} = 13.3$ $\quad$ $t_{134} = 13.4$

$d_{133} = 299.291$ $\quad$ $d_{134} = 303.777$

$v_{133} = 44.857$ $\quad$ $v_{134} = 45.184$

Velocity as a function of distance, with 40 ft/sec illustrated

As can be seen from the graph above, the barrel hits bottom in a little over 13 seconds with a velocity of approximately 45 feet per second; too fast for the Atomic Energy Commission, which put an end to dumping of nuclear waste in the Atlantic Ocean.

Analytic Solution

Later in the course we can solve $\frac{\mathrm{dv}}{\mathrm{dt}}=\frac{32.2}{527}\cdot\left(57-0.08\ \mathrm{v}\right)$ analytically using separation of variables. We often say that we want analytic solutions if we can get them, and settle for approximate numerical solutions if we have to.

If $\frac{dv}{dt}=k\cdot\left(a-b\,v\right)$, then $\int\frac{dv}{a-bv}dv=\int k\,dt$, and $-\frac{1}{b}\ln\left|a-bv\right|=kt+C$, so $\left|a-bv\right|=Ae^{-bkt}$.

Since $a-bv>0$, we have $a-bv=Ae^{-bkt}$ and $v\left(t\right)=\frac{-Ae^{-bkt}+a}{b}$. For our values of *a*, *b*, and *k*, we have $a=57$, $b=0.08$, and $k=\frac{32.3}{527}$. Since the initial velocity is zero, we know that $B=712.5$. So, our final model is $v\left(t\right)=-712.5e^{-0.004888t}+712.5$.

As before, this function really doesn't answer our question. We can tell how fast the barrel is traveling with respect to time, but we need to know the relationship between velocity and distance.

So, $h\left(t\right)=\int-712.5e^{-0.004888t}+712.5\,dt=145765.139e^{-0.004888t}+712.5t+c$.

Since $h\left(0\right)=0$, we have $h\left(t\right)=145765.139e^{-0.004888t}+712.5t-145765.139$.

We now have analytic solutions for $v\left(t\right)=-712.5e^{-0.004888t}+712.5$ and $h\left(t\right)=145765.139e^{-0.004888t}+712.5t-145765.139$. Unfortunately, our rules of algebra are not too helpful in finding *v* as a function of *h*. The function *h*, above, is a combination of a transcendental function and an algebraic function. We could solve for *h* in terms of *v*, but since we only want to know the velocity when $h=300$, we will refrain from that bit of algebra for now. We can solve $300=145763.781e^{-0.004888t}+712.5t-145763.781$ numerically to find $h=300$ feet when $t=13.266$ seconds. So, $v\left(13.266\right)=44.735$ ft/sec. Our Euler's method solution approximated the velocity at around 45 ft/sec.

Analytic vs. Numerical Solutions

It is fair to ask, "What advantage does the analytic solution have over the numerical solution?" After all, we need a numerical solution (zero) anyway to find the appropriate value of *t*. Often, the closed form of an equation gives us some useful information that is not obvious from the differential equation. In this case, little additional information is gathered from the analytic solutions.

If we solved further to find $h\left(v\right)=-145763.781\left(0.0014035v+\ln\left(1-0.0014035v\right)\right)$,

we could determine the depth when the velocity is 40 ft/sec. We have $h\left(40\right)=238.678$, so the barrel is moving too fast at 300 feet. What has been gained in doing the extra algebra? Does $h\left(v\right)=-145763.781\left(0.0014035v+\ln\left(1-0.0014035v\right)\right)$ tell us something that we couldn't find using only Euler's method?

In this case, the answer (at least to me) is no. The analytic solution doesn't give us any insight into the solution that is missing from the numerical work of Euler's Method. Unfortunately, you don't often know if the extra analytic work will be useful until after you have done it!

Reference: Braun, Martin. *Differential Equations and Their Applications*, 3rd Ed., Springer-Verlag New York, Inc., New York, 1983

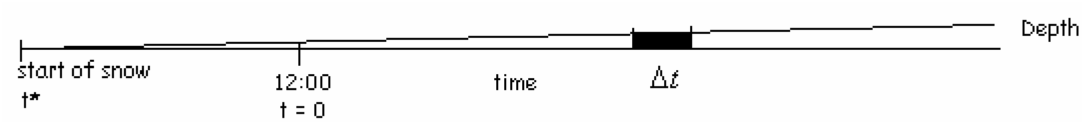
The Snow Plow Problem (Challenge problem for After the Exam)

Dan Teague, NCSSM

Snow starts to fall before noon and falls at a constant rate all day. At noon, a snow plow starts to clear a road. The velocity of the snow plow is such that it removes a constant volume of snow per unit time. The plow goes 1 mile during the first hour and one-half mile during the second hour. What time (to the nearest minute) did it start to snow?

At first reading, this problem seems unsolvable with the given information. However, we can set up some differential equations that allow us to arrive at a nice solution.

Solution: We know that the snow falls at a constant rate, so the depth, *D*, of the snow is given by $\frac{dD}{dt} = k$. The volume of snow is being removed at a constant rate, so $\frac{dV}{dt} = r$. Over time, the depth of the snow increases, slowing down the plow. The volume of snow plowed in some time interval Δt depends on the product of the depth of the snow and the distance the plow has moved, Δs.



Now, we know that $\Delta V \doteq D(t)\Delta s$, so that $\frac{\Delta V}{\Delta t} \doteq D(t)\frac{\Delta s}{\Delta t}$. This gives us the differential equation $\frac{dV}{dt} = D(t)\frac{ds}{dt}$. Also, since $\frac{dD}{dt} = k$, we know that $D(t) = kt + c$ with *c* being the depth of snow at noon ($t = 0$). We also know that the snow began to fall when $D(t^*) = 0$, so $t^* = \frac{-c}{k}$.

Putting this together, we can rewrite $\frac{dV}{dt} = D(t)\frac{ds}{dt}$ as $r = (kt + c)\frac{ds}{dt}$, so $\frac{ds}{dt} = \frac{r}{(kt + c)}$. This is a separable equation, so $\int ds = \int \frac{r}{(kt + c)}dt$ and $s(t) = \int_0^t \frac{r}{kx + c}dx$.

But we know that the distance in the first hour is 1 mile, so $1 = \int_0^1 \frac{r}{kx + c}dx$. Integrating, we find $1 = \frac{r}{k}\ln\left(\frac{k + c}{c}\right)$. But since $t^* = \frac{-c}{k}$, we have $1 = \frac{r}{k}\ln\left(1 - \frac{1}{t^*}\right)$.

We also have $\frac{1}{2} = \int_1^2 \frac{r}{kx + c}dx$ which gives us $\frac{1}{2} = \frac{r}{k}\ln\left(\frac{2k + c}{k + c}\right)$. Unfortunately, we can't write this easily in terms of $t^* = \frac{-c}{k}$. A better choice would be $\frac{3}{2} = \int_0^2 \frac{r}{kx + c}dx$, so $\frac{3}{2} = \frac{r}{k}\ln\left(\frac{2k + c}{c}\right) = \frac{r}{k}\ln\left(1 - \frac{2}{t^*}\right)$. Now, we have a system of two equations in t^*.

If $1=\frac{r}{k}\ln\left(1-\frac{1}{t^*}\right)$ and $\frac{3}{2}=\frac{r}{k}\ln\left(1-\frac{2}{t^*}\right)$, then dividing the second by the first, we have $\frac{3}{2}=\frac{\ln\left(1-\frac{2}{t^*}\right)}{\ln\left(1-\frac{1}{t^*}\right)}$, or $3\ln\left(1-\frac{1}{t^*}\right)=2\ln\left(1-\frac{2}{t^*}\right)$. Solving for t^*, we find that $\left(1-\frac{1}{t^*}\right)^3=\left(1-\frac{2}{t^*}\right)^2$ or $1-\frac{3}{t}+\frac{3}{t^2}-\frac{1}{t^3}=1-\frac{4}{t}+\frac{4}{t^2}$ (where t is our t^*), so $\frac{1}{t}-\frac{1}{t^2}-\frac{1}{t^3}=0$. This is equivalent to $t^2-t-1=0$, so $t^*=\frac{1\pm\sqrt{5}}{2}$, the golden ratio (you were probably suspecting e or π). Since t^* is negative (the snow starts before noon), we have $t^*=\frac{1-\sqrt{5}}{2}\approx-0.618$. This is about 37 minutes before noon. So, to the nearest minute, the snow began at 11:23 am..

Explorations

Susan Wildstrom has "explorations" and other smallish stand-alone presentations that she uses in both precalculus and calculus. They are mostly all there at her website (see below). If any of them would be useful to share with your colleagues, feel free to download them and include them (with credit to her, please, since anything that she puts on her website is material that she has developed). Included at the precalculus page (they are in the assignments section of each page) is a document on a way to teach mathematical induction that is a bit different than the usual book presentation (and works very well with high school students). In the calculus area there are at least three or four "explorations" that allow students to discover derivative rules - product and quotient, exponential and logarithmic, and chain rule. If you decide to use any of her things she requests that you 1) let her know in advance what you will be using and 2) send her a copy of the newsletter in which they appear. If you need WORD documents (rather than the PDF form in which they come up on the website) let her know and she can send you those as attachments.
Susan Schwartz Wildstrom, Walt Whitman High School, Bethesda, MD www.wildstrom.com

Miscellaneous Information

1. Look at the Sketchpad activities based on the 1st chapter of Paul Foerster's text including the door problem. They are available for free at
http://www.keypress.com/sketchpad/general_resources/classroom_activities/calculus_pdc2003/

2. The Department of Mathematics at UC-Berkeley offers students an online calculus placement test that they may use to decide between the different math courses offered. You may find it useful. It is only 24 multiple choice questions and can be generated in various versions. A fairly complete analysis of the results is obtained, including degree of difficulty and topic of questions.
http://math.berkeley.edu/courses_placement.html

3. **From Mike May,** S.J., Chair, Department of Mathematics and Computer Science
Saint Louis University, maymk@slu.edu
A collection of java applets useful for teaching calculus. The easiest URL for the applets is
http://www.slu.edu/classes/maymk
(That redirects to another file with a longer URL that is harder to remember.) The collection includes about 35 applets and is organized to follow a traditional syllabus. There is some duplication as the applets come from several sources.
http://www.slu.edu/classes/maymk/Riemann/Riemann.html
and the secant-tangent line progression
http://www.slu.edu/classes/maymk/SecantTangent/SecantTangent.html
visually looking at sums and sequences is also interesting
http://www.slu.edu/classes/maymk/SeqSeries/SeqSeries.html

He would like to impose two conditions on use of the material:
1) If you reproduce the material locally, please give appropriate attribution;
2) He thinks the web site would be strengthened if he had more pedagogical material to go along with the applets. In particular, he would like sample worksheets/assignments/ lesson plans to go with the applets. If you use the material please send him a copy of material along this line that you produce.

4. Newest edition of Finney Demana Kennedy book. From the website, it looks fabulous! http://www.phschool.com/fdwk/

5. From Stu Schwartz, Wissahickon High
   Terrific applet written by Marek Rychlik
   http://alamos.math.arizona.edu/ODEApplet/JOdeApplet.html.
   If you have a projector in your classroom, this applet allows students to see how slope fields are created and changed with as many specific solutions as you want, creating a Table using Euler or other methods. Even if you don't have a projector, you can give the students the applet.

   I walked them through the rabbit problem and the free fall problem in class and then asked them to do the escape velocity problem at home. It engendered a lot of discussion and for the first time, I felt that I had really taught slope fields well and that the combination of creative problems and intelligent use of technology created a super learning environment.

6. Go to www.ticalc.org and search for slope fields and download the program with ti-connect.

7. Here is a link to the Key Curriculum Press page: http://www.keypress.com/catalog/products/supplementals/Prod_CalcExpl.html

8. APEX class tools www.apexvs.com It is a complete online course for AP Calculus, but can use parts of it as supplementary material. It costs about $30 per student per year. The students can use it from home. It has quizzes and diagnostic tests that the instructor can use to monitor progress.

9. Suggestion for test format: 5 multiple choice with calculator, 5 without calculator; 2 free response questions with calculator, 1 without. It mirrors the AP exam and by May the kids are comfortable with this format. Go to the AP website for old free response questions and get one or more of the practice books available for question ideas.

10. Take a look at www.hotmath.com

11. Poster of over 1400 digits of epsilon! http://www.mathteacherstore.com/middle/midlpost/5-8/epsilon.htm

12. Meg Clemens, Canton Central, Canton NY has PowerPoint slides on her web site http://www.ccsdk12.org/~mclemens/STEM/Fraction%20Infractions.ppt that cover many common errors. The slides are large: 24" by 24". I use them to print posters for the classroom on a wide printer.

13. Worksheet online called "Algebraic Atrocities" URL is long, but just Google that phrase and it's the first hit. Calculus For Dummies has nice little sections called "Things to Remember" and "Things to Forget"

14. This is great!!! http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html#ablpuzzle
    If you haven't found the above website yet - it is a very neat way to practice matching graphs with their derivatives! Cleverly done.

15. The gallery http://www.univie.ac.at/future.media/moe/galerie.html gives a bunch of activities for a number of topics.

16. Another one is here: http://wims.unice.fr/wims/wims.cgi?session=7R37AED85C.2&+lang=en+module=U1%2Fanalysis%2Fderdraw.en The students can actually draw it themselves and check their answers.

17. Teachers should look at merlot.org, a clearing house for applets and other stuff (for all sorts of math, and lots of other subjects). The advantage and disadvantage is that the sites from there that you'll find links to have been reviewed. Mark Howell, Gonzaga High School, Washington, DC

18. Bruce Kahn, Smithtown HS Western Campus, Smithtown, NY 11787, recommends these sites:
    www.bkahn.us
    http://www.ies.co.jp/math/products/calc/menu.html
    http://www.ies.co.jp/math/java/
    http://clem.mscd.edu/~talmanl/MathAnim.html
    http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/
    http://www.math.dartmouth.edu/~klbooksite/all_applets.htm
    http://astro.ocis.temple.edu/~dhill001/relatedrates/relatedrates.html
    http://www.teacherlink.org/content/math/interactive/flash/home.html
    www.calculus-help.com. Click on tutorials and view the one on chain rule.
    Thanks to W. Michael Kelley for doing this great site.

19. Calculus Webpage with Lessons and assignments
    http://www.pen.k12.va.us/Div/Winchester/jhhs/math/lessons/mcalc.html

20. An opportunity for AP calculus community. It is directly aimed at people teaching multivariable calculus. (While multivariable calculus is not an AP course, there are a number of high schools where the math teachers find themselves teaching multivariable calculus.)

    Every year the Mathematical Association of America (MAA) sponsors a number of Professional Enhancement Programs (PREP workshops). Among this year's workshops http://www.maa.org/prep/2006/ is an online workshop.

    This workshop looks at using the computer algebra system Maple as a tool for teaching multivariable calculus. http://euler.slu.edu/GrantWebPages/PREP06Calc3Maple/index.html The techniques can be used in a variety of courses, but to make the discussion concrete they will focus on calculus III this summer. Participants will need a high speed internet connection and a computer.

Those Sign Chart Issues Again
David Bressoud, AP Calculus Test Development Committee

A sign chart, by itself, is not sufficient justification for anything. I have retired from the Development Committee and will not be at the Reading this coming summer, so things may change, and what I say should not be taken as official policy. The sense of the Committee when I left it this past summer was that the description in words must be able to stand on its own as sufficient justification.

I want to take this opportunity to alert teachers to a common mistake that we see. Students often "justify" that a function has a local maximum at a particular value of x by stating that the function is increasing to the left of that value and decreasing to the right. This is considered to be a definition, not a justification.

A justification should be in terms of the behavior of the derivative: The derivative of f is positive to the left of this value and negative to the right. Or it could be that the derivative is zero at that value and the second derivative is negative over an interval that contains that value. The implication that we are willing to read into either statement is that this information about the derivatives implies that the function is increasing to the left and decreasing to the right and thus the definition of a local maximum is met.

Riemann sum program for ti83+

http://www.dudefree.com/Technology/TI/Programs/SHOWRAM.php
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BOOKS FOR THE LIBRARY

The list below is a very good one. Prime Obsession is non-trivial and will require work from a good HS students (but it is great as summer reading for teachers!) Three omissions: Journey Through Genius, Euler: The Master of Us All, both by Wm. Dunham and An Imaginary Tale by Nahin. "In Code" was written by a 17 year old.

Flatland	Edwin Abbott
Math and the Mona Lisa	Bulent Atalay
Tour of the Calculus	David Berlinski
Sphereland	Dionys Burger
Prime Obsession	John Derbyshire
Uncle Petros & Goldbach's Conjecture	Apostolos Doxiadis
Number Devil: A mathematical adventure	Hans Enzensgerger
In Code: A Young Womans' Mathematical Journey	Sarah Flannery
Imaginary Numbers: An anthology	William Frucht
Chaos	James Gleick
Parrot's Theorem	Denis Guedj
Curious Incident of the Dog in the Night-time	Mark Haddon
The Man Who Loved Only Numbers	Paul Hoffman
Political Numeracy	Michael Meyerson
Mathematician Plays the Stock Market	John Allen Paulos
Innumeracy	John Allen Paulos
Once Upon a Number	John Allen Paulos
Fragments of Infinity	Ivars Peterson
Mathematical Tourist	Ivars Peterson
Jungles of Randomness	Ivars Peterson
Flatterland	Ian Stewart
Man Who Counted	Malba Tahan
A Beginner's Guide to Constructing the Universe	Michael Schneider
Mathematical Scandals	Theoni Pappas
Fourth Dimension	Rudy Rucker

Calculus Worksheet for the Summer (They give them summer readings in English!)
Adrienne Hestenes, Xavier College Prep, Phoenix, AZ

AB/BC CALCULUS SUMMER WORK

I. Calculator Basics

1) Determine which of the following gives a complete graph for the indicated equation:

a) $y = -x^3 + 8x^2 - x + 5$

(i) [-10,10] x [-10,10]
(ii) [0,10] x [-10,80]
(iii) [-5,10] x [-10,80]
(iv) [-50,50] x [-100,100]

b) $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$ ***

(i) [-10,10] x [-10,10]
(ii) [-2,20] x [-20,20]
(iii) [0,20] x [-5,5]
(iv) [-5,20] x [-5,5]

Not even these windows will catch a subtlety only calculus can find for us!

2) Estimate the largest and smallest values of each of the following functions on their given intervals.

a) $f(x) = 2^x + x^2$ [-4,1]

b) $y = (\cos x)^x$ [-1.5,4.75]

c) #1b) [2,4]

d) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$ (-2,2)

3) Graph $y = \dfrac{x^2 - 9}{x - 3}$ on your TI-83.

a) Explain why this graphs is a linear function instead of a rational function having a vertical asymptote at $x = 3$.
b) Zoom in on the graph around x = 3. Give the viewing window when you first see the empty pixel at x = 3 **or** you notice the graph getting "jagged" around there. (The newness of your calculator will determine which of these pictures you get.)
c) If you had to fill in a y-value when x = 3, what would it be?

4) An open box is to be made from cutting squares of side *s* from each corner of a piece of cardboard 25" by 30".

a) Write an expression for the volume, *V*, of the box in terms of *s*.
b) Draw a graph of $V(s)$. You may use your grapher. Identify the domain and range of this graph.
c) What domain and range make sense in this problem situation? Highlight this on your original graph.
d) Find the value of *s* that will give the maximum volume. What is the maximum volume?
e) What value(s) of *s* will give a volume of 1225 cubic units?

II. Cartesian Plane Basics

5) Determine the slope, the length, and the midpoint of the segment containing (1,-2) and (3,2).

6) For what values of *k* is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what values of *k* are the two lines perpendicular?

7) Plot the line $2x - 5y = 10$ indicating x- and y- intercepts. Please be sure to label your axis to indicate your scale.

8) Find the distance from point P(1,2) to line L: $x+2y=3$.

9) You need a Lear jet for one day. Knowing that Swissair rents a Lear jet with a pilot for $2000 a day and $1.75 per mile, while Air France rents a Lear jet with a pilot for $1500 a day and $2.00 a mile, find the following:
a) For each company, write a formula giving cost as a function of distance traveled.
b) Sketch graphs of both functions, labeling intercepts and point of intersection.
c) If cost were the only issue, when would you rent from Air France?

10) Find the line that passes through (-1,3) and the point of intersection of the lines $x+3y=1$ and $2x-y=-5$. Leave your answer in point slope form.

III. TOOLKIT Functions

You should be very familiar with the following functions and be able to envision them in your mind without the advantage of a graphing calculator:

1. constant function: $f(x)=k$, k is a constant
2. linear function: $f(x)=mx+b$
3. identity function: $f(x)=x$
4. absolute value function: $f(x)=|x|$
5. piece-wise (or partitioned domain) function: $f(x)=\begin{cases}\sin x, \text{ when } x\le -1\\ x-3, \text{ when x>-1}\end{cases}$
6. greatest integer function: $f(x)=[x]$
7. quadratic function: $f(x)=ax^2+bx+c$
8. polynomial function: $f(x)=ax^n+bx^{n-1}+...+mx+n$
9. radical function: $f(x)=\sqrt{x}$
10. rational function: $f(x)=\dfrac{ax-b}{cx-d}$
11. exponential function: $f(x)=b^x$ where $b>0,\ \neq 1$
12. logarithmic function: $f(x)=\log_b x$

11) Identify the domain and range of the following:

a) $f(x)=\begin{cases}4-x^2, & -5<x<-1\\ \dfrac{3}{2}x+\dfrac{3}{2}, & -1\le x\le 3\\ x+3, & 3<x\le 10\end{cases}$

b) $g(x)=-2[2x]+1$

c) $h(x)=-2+5\sqrt{4-x}$

d) $f(x)=\dfrac{x}{x^2-1}$

12) Describe how each graph can be obtained from the graph of
$f(x)=\sqrt{x},\ g(x)=\dfrac{1}{x},\ \text{h(x)}=|\text{x}|,\ \text{k(x)=x}^3,\ \text{l(x)}=\log x,$ or $m(x)=2^x$.

a) $y=.5(x-4)^3+2$

b) $y=|x+2|-1$

c) $y=-\dfrac{7}{x+1}-3$

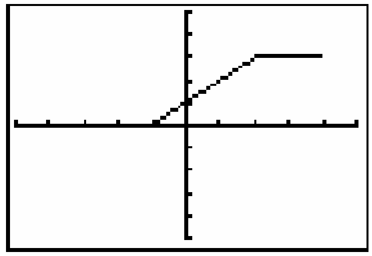
d) $y=-\dfrac{5}{2}\sqrt{3-x}+4$

e) $y=2\log(x+3)-5$

f) $y=2^{(1-x)}-3$

13) Find the inverse of $f(x) = -(x+3)^2$ when $x \geq -3$. Graph both functions on the plane and analytically prove $f(f^{-1}(x))$.

14) Given the following graph of $f(x)$:



Sketch a graph of the following functions:

a) $f(x)+2$

b) $-f(x)+1$

c) $1 + f(x + 3)$

d) $3f(x-2)+1$

15) Identify the following functions as odd, even or neither:

a) $y = x^4$

b) $y = x - x^4$

c) $y = \dfrac{1}{x^2 - 4}$

d) $y = -2x^3 + 4x$

IV. Parametric Equations

16) Graph $\begin{matrix} x(t) = 3t \\ y(t) = 2 - 2t \end{matrix}$ on [0,1]. (Use your calculator.) Write a Cartesian equation that models this graph. Identify the initial and terminal points.

17) Follow the directions in #16) with the parametric equations $\begin{matrix} x(t) = 2t - 1 \\ y(t) = t + 1 \end{matrix}$ on [0,2].

18) Parametric equations give us the ability to graph curves that are **not** functions. Graph the following two sets of curves, determine their corresponding Cartesian equations, and tell how you can determine which direction (up/down **or** left/right) the parabola will open based on their parametric equations.

a) $\begin{matrix} x(t) = \dfrac{t^2}{2} \\ y(t) = t \end{matrix}$ on $[-2,2]$

b) $\begin{matrix} x(t) = \sqrt{t-1} \\ y(t) = t - 1 \end{matrix}$ on [1,5]

19) Graph $x = y^2 - 6y + 11$

a) in function mode using two functions (HINT: complete the square)

b) in parametric mode (You need to come up with the parametric equations.)

20) Graph and identify the kind of conic the graph represents:

a) $\begin{matrix} x(t) = 3\cos t \\ y(t) = 3\sin t \end{matrix}$ on $[0, 2\pi]$

b) $\begin{matrix} x(t) = 2\cos t \\ y(t) = 5\sin t \end{matrix}$ on $[0, 2\pi]$

V. Trig Review

Your life will be a lot easier next year if you can remember your trig values for special angles, the basic shape of the six trig functions, their domain and range, and these identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

Your life this year will be a lot easier if you can do these WITHOUT a calculator!!! *We always use* ***radians*** *and* ***never*** *work with degrees.*

21) Evaluate:

a) $\sin\pi$ b) $\cos\left(\frac{3\pi}{2}\right)$ c) $\tan\left(\frac{5\pi}{6}\right)$ d) $\sin\left(\frac{4\pi}{3}\right)$

e) $\cos\left(\frac{7\pi}{4}\right)$ f) $\sec\left(\frac{2\pi}{3}\right)$ g) $\cot\left(\frac{5\pi}{4}\right)$ h) $\csc\left(\frac{11\pi}{6}\right)$

i) $\sin\left(-\frac{27\pi}{6}\right)$ j) $\sin\left(-\frac{17\pi}{3}\right)$ k) $\sin\left(\frac{108\pi}{8}\right)$ l) $\cos\left(-\frac{11\pi}{4}\right)$

22) Change the following to radians:

a) 510° b) 120° c) 135° d) -210° e) -315°

23) Identify which of the six trig functions are

a) odd
b) even
c) neither

24) Which two equations have the same graph?

a) $y = \sin x$ b) $y = \sin(-x)$ c) $y = \cos(x)$ d) $y = -\sin(x)$ e) $y = -\cos(x)$

25) Consider the function $y = \sqrt{\dfrac{(1+\cos 2x)}{2}}$.

a) Can x take on any real value?
b) How large can $\cos 2x$ become? How small?
c) How large can $\dfrac{1+\cos 2x}{2}$ become? How small?
d) What are the domain and range of the original function?

26) Describe the angle in radians:

a) $\frac{1}{8}$ revolution clockwise b) $1\frac{1}{3}$ revolutions counterclockwise

c) $1\frac{3}{4}$ revolutions counterclockwise d) $2\frac{1}{2}$ revolutions clockwise

27) Under the given conditions, find $\sin\theta$ or $\cos\theta$, whichever is not given, and state the quadrant θ lies in.

a) $\cos\theta = \frac{3}{5}$, $\sin\theta > 0$ b) $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta < 0$

c) $\sin\theta = -\frac{12}{13}$, $\cos\theta > 0$ d) $\cos\theta = \frac{\sqrt{5}}{5}$, $\sin\theta > 0$

28) Simplify:

a) $\cos\frac{2\pi}{3}\cdot\cos\frac{\pi}{6}+\sin\frac{2\pi}{3}\sin\frac{\pi}{6}$

b) $\cos\frac{2\pi}{3}\cdot\sin\frac{\pi}{6}+\cos\frac{\pi}{6}\cdot\sin\frac{2\pi}{3}$

29) Evaluate:

a) $\sin\left(\arccos\frac{1}{3}\right)$

b) $\cos\left(\arcsin\left(-\frac{1}{4}\right)\right)$

c) $\tan\left(\arcsin\frac{1}{2}\right)$

d) $\sec\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

30) Arrange these in order from least to greatest:

a) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$, $\arccos 0$, $\arccos\left(\frac{1}{2}\right)$

b) $\arctan\left(-\sqrt{3}\right)$, $\arctan 0$, $\arctan\left(\frac{1}{2}\right)$

31) Find the value of each:

a) $\sin\left(\arcsin\frac{1}{2}+\arccos\frac{\sqrt{3}}{2}\right)$

b) $\cos\left(\arcsin\frac{1}{2}+\arccos\frac{\sqrt{3}}{2}\right)$

c) $\cos\left(2\arcsin\frac{1}{5}\right)$

d) $\tan\left(\arctan\sqrt{3}+\frac{\pi}{3}\right)$