The North Carolina Association Of Advanced Placement Mathematics Teachers Newsletter

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Web Address
www.ncaapmt.org/calculus

Notes From The President's Desk

I want to thank those of you who attended the NCAAPMT session here in Greensboro back in October. Special thanks to those who prepared to present some of the 2004 free response questions and standards: Rhea Caldwell, Stephen Davis, Gloria Dupree, and Rona Schriber.

As I sat home this weekend wondering how to make up for lost time due to "weather" situations, I thought about the "snow days" of my calculus preparation. I attended a small liberal arts college. In January of my freshman year, the board of trustees announced that tuition for the following year would increase from $1500 a semester to $2500. Percent of increase aside, by today's standards that's a good deal! However, in the winter of 1970 many parents and students were infuriated at the amount of the increase. The student body thought they'd teach the college a lesson and we went out on strike on March 1st. That's right – "no more teachers, no more books, etc." After a month of living on campus without a real good reason, my school as well as a great majority of college campuses joined in the Vietnam Moratorium on April 1st. The end result was that we were required to take exams on material we learned on our own or in study groups and were given grades of pass or fail. The real shocker came the following fall when, in calc II, I was expected to know several different methods of integration without adequately understanding the foundation of the integral. It wasn't until I had been teaching calculus (many years after 1970!) that I started to understand it and develop an appreciation for its power and significance.

Had I been a student in this current age of technology, I might not have stayed so ignorant for so long! We have so many resources at our disposal. AP Central is a wealth of information. Course descriptions, sample syllabi, exam information, previous free response questions, tips for teaching, Ben Klein's Question of the Month are only a few of the many sources you'll find there. Through AP Central, our Chief Reader, Karen Diefenderfer, will present a mini workshop on the Fundamental Theorem of Calculus on March 15th and 16th from 6:30 – 8:30 each evening. From Gaston Caperton, president of the College Board, the first Advanced Placement Report to the Nation was released on January 25. "The report uses a combination of state, national, and AP Program data in new ways to provide each U.S. state with a context for celebrating its successes, understanding its unique challenges, and setting meaningful and data-driven goals to connect more students to college success."

The calculus electronic discussion group is a knowledgeable and supportive group of colleagues that are only a fingertip away. If you are not a member, you can access it through AP Central to sign up. There are going to be some special sessions for AP Calculus, AP Statistics, and Math Club/Team leaders at the Southeastern Region MAA meeting at Meredith College in Raleigh on March 11-12. The special sessions will be held on Saturday, March 12 from 2:30 – 5. Dan Teague (**teague@ncssm.edu**) of the North Carolina School of Science and Mathematics is organizing a workshop for all who are interested in advanced high school mathematics. Contact Dan with expressions of interest and for further details.

Please remember to renew your membership, $5.00 a year, to receive the two newsletters. You can send your check, payable to NCA PMT, to our treasurer Jeff Lucia, 718 Landsdowne Road, Charlotte, NC 28270. His email address is **jeff.lucia@providence.org**. Good luck getting it all done and teaching your students not to use pronouns in explanations as well as adequately describing those sign lines! We always welcome your comments and look forward to helping you in any way.

Trish Morris

From the Secretary's Desk

Three months (August to November) spent at home with a broken leg and ankle gave me time to explore wheel chair life and get my coursework done for my doctoral program. Now, I am back and things are going well. Luckily, the kids in AP Physics/AP AB Calculus and my BC Calculus course had a substitute who kept the pace. They did not know AP test strategy but they did know content. I hope you enjoy this newsletter and are able to use some of the valuable information here. Please email any thing you think might be helpful to other calculus teachers. My course work has taught me a lot about writing with math symbols and using technology better so I am grateful for that. But, I never have enough time to get everything done that I would like for all you guys out there who encourage me so much. Please forgive if we have made any mistakes.

Deborah Britt **deborah.britt@asheville.k12.nc.us**

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NCA^{2}PMT

Minutes of Annual Business Meeting

Thursday, October 7, 2004
NCCTM Conference
Koury Convention Center
Greensboro, NC

Trish Morris, President, opened the meeting by introducing the board members who were in attendance. Those introduced were: Jeff Lucia, Gloria Dupree, Ben Klein, Steve Davis, Ed Tharrington, Ray Jernigan, Sue Wall, Deborah Britt, and Emogene Kernodle.

Jeff Lucia gave the treasurer's report. There are 370 paid memberships with an additional 60 who have dues in arrears for a total of 430 in the database. NC, 40 other states, and 5 foreign countries are represented in the membership. The summer newsletter is paid for and we have a little under $2000 in the treasury. We have secured a grant from CMSTE (Center for Mathematics, Science and Technology Education) with the help of Dr. David Royster at UNCC that will fund 25% of the cost of the newsletter which will add to our account resulting in around $2300.

2004 Questions were presented as follows:
AB1/BC1 Rhea Caldwell (Traffic flow), AB 6 (first AB slopefield problem) Steve Davis, BC 6 Gloria Dupree (BC Taylor Polynomial and Lagrange Error)

Ben Klein reported on MAA's placing some new emphasis on AP teachers. Dan Teague at NCSSM sent a message about SIGMAA and the meeting being planned for Meredith College in Raleigh on March 11-12, 2005.

There was some discussion about the new policy of College Board that sign charts by themselves without written explanation will not be acceptable for credit for reasons on the AP exam. It was noted that everyone should read the sign chart rationale and examples on the AP Central website.

Ben gave the list of Calculus Test Development committee members: David Bressoud, Chairman, Janet Beery, David Loman, Carol Miller. New members who just had their first meeting are Guy Mauldin and Monique Morton. Ben's term on the committee is now over. He states that the committee would continue to emphasize more writing. He also stated that the committee was not informed prior to the announcement at the AP Reading that their may be a 2 week window to take the exam coming – in other words, there would be a testing date on 2 different days. Since the committee already has to make out two versions of AB, two versions of BC and the alternate exams, it would be difficult to add additional tests. Another problem is that the test is designed over a 2 year period. So, we will await further official word on this.

Several web addresses were given:
http://www.ncaapmt.org/calculus
http://ap.central.collegeboard.com
http://www.maa.org/southeastern
http://www.maa.org

Trish reported that on-line scoring would be out for 2005 but may be used again in 2006.
The AP Reading will be June 2-8, 2005 in Ft. Collins, CO with June 1 and 9 as travel days. The 2003 Multiple Choice will be released this month (October). It was noted that according to recent article at AP Central that _ of first semester calculus will taught at high school level by 2006.

Audience questions covered the two window testing (which may happen in 2 years) issue and possible January test dates (which will not happen).

Respectfully submitted by Deborah Britt, Executive Secretary

An Introduction to our new Chief Reader: Caren Diefenderfer

By Ben Klein, Davidson College

It's been said that the more things change, the more they stay the same. The biographical data for the current and previous Chief Readers for Calculus certainly bears this out. Let's note some of the unexpected similarities between Caren and Larry Riddle, the previous Chief Reader. Each was raised in Pennsylvania, each is musical, and each is a respected teacher at a women's college in the South. Then there are the expected similarities: both have long and distinguished histories with the AP Calculus Program, and both have doctorates in mathematics.

Of course, there are differences between Larry and Caren. Larry is an instrumentalist while Caren is a vocalist. Larry did his graduate work in the mid-west while Caren did hers in sunny California. And, if you saw them side-by-side, you would notice that Larry is a good bit taller than Caren.

Now, having established that Caren is not Larry's clone, I will now tell you a little about her, and then I will let her tell you a little about herself.

Her Pennsylvania upbringing notwithstanding, Caren has family roots in the "Old North State." Her maternal grandmother grew up near Newton and was related to North Carolina icon, Senator Sam Ervin. On top of that, her husband, David, a real estate appraiser and, according to Caren, "baseball coach extraordinaire," is a Virginian. Caren has two boys, Mark and Joseph, and, in her words: "Mark is 13, entering 8th grade and is an avid baseball player and fan. Joseph is 7, entering 2nd grade and is a live wire who tries to do everything his big brother does."

Caren is currently a member of the Department of Mathematics at Hollins University. Hollins, by the way, is primarily a liberal arts college for women located in the western part of Virginia; Roanoke, to be precise. Any one who has been in that part of Virginia will tell you that it is one of the most beautiful parts of what is already a beautiful state.

At Hollins, Caren has served as Department Chair and as Chair of the Division of Mathematical and Natural Sciences. It's clear that the folks at Hollins recognize talent when they see it. Among other interests and accomplishments, Caren is a leader at Hollins and indeed nationally in quantitative literacy and reasoning.

Caren started her undergraduate work at Smith College but transferred to Dartmouth after two years. She graduated, summa cum laude, with a major in mathematics from Dartmouth and then went on to graduate study at UC-Santa Barbara where she worked in functions of several variables and earned her Ph.D. She actually started teaching at Hollins before she received her Ph.D. and has now been a member of the Hollins Faculty for twenty-seven years.

Her involvement with AP Calculus began in 1968-69 when she took AB Calculus and earned (what else) a five on the exam. Not counting that auspicious beginning, she has a sixteen-year history at the AP Calculus Reading and did a superb job this June in her first year as Chief Reader.

Now, as promised, Caren will tell you something about herself. I have excerpted the following from a "Personal Mathematics Biography" that she sent me. The parenthetical remarks at the end of the paragraphs are mine.

> My earliest mathematical recollection is one of infinity. As a young child, I remember having a book at my grandparents' house that had a picture of a bear reading a book on the cover. As I looked at the book that the bear was reading, I could tell that book cover had another picture of a bear reading a book. At the time, I wondered how many bears reading a book were contained in that picture and realized that it could go on forever. I remember keeping this thought to myself because I didn't think my grandparents or my parents would want to answer that question. [Contemplation

of infinity is a particularly appropriate activity for a Chief Reader; the folders do seem to go on forever and ever.]

> Another early memory I have occurred when I was in 2nd grade. I was taking a standardized test, although I was the only student in a room with the tester. One part of the test included rows of making a pattern. I noticed that I could work this pattern horizontally or vertically and started completing the pattern as quickly and accurately as I could. The tester was very unhappy with me and insisted that I work horizontally and left to right only. This was my first experience with feeling that someone disliked a creative solution that I discovered. [Now that Caren is the Chief Reader, it's her job to see that students <u>follow the instructions</u>.]
>
> I have an older sister who was three years ahead of me in school. Her entire Algebra II class was having trouble understanding their teacher, my sister would come home from school and ask my dad for help, after dinner she'd share the information with two or three friends via telephone and then they'd have an informal phone tree to help everyone complete the homework for that night. As in most families, we had several "Dinner Rules" and one of them had always been "No singing at the dinner table." That year, my mom imposed a new rule and it was "No math books at the dinner table." My mom believed dinner was the time for family conversation and wanted everyone to participate. I've often wondered if there is another family that banned math books from their dinner table.
>
> When I was a senior in high school and taking calculus, I would sometimes ask my dad for help. He had the following modus operandi:
>
> - First, he wanted me to explain the problem to him. Of course, if I could explain the problem to him, while giving him the explanation I could usually also figure out how to do the problem myself.
> - The second step was that he'd look at me and say "Think" and we'd sit in silence for several minutes.
> - Thirdly, he'd give me a hint without doing the problem for me.
>
> One night, we went through the first two steps and he acknowledged that he didn't know how to proceed and couldn't give me a hint. I clearly remember the moment and thought to myself, "I must be getting smart!" My dad then suggested that we go to his office, where he had his college calculus book, because he thought if he looked at his own book, he might remember how to solve this particular problem. Driving to the office with my dad that night, I remember feeling very satisfied that I was learning something that my dad had to look up in a book. When we got to the office and he found his book, he was able to give me a hint so I could complete my assignment.

Caren concluded her Biography with the following.

> … my AP colleagues are now another important group in my mathematical journey. I have formed many strong friendships – both personal and professional – during my 16 years at the AP Calculus Reading. It has been the most important and satisfying professional experience of my career … . Thanks, AP, for being good to and for me!

And we who have been lucky enough to work with her would echo "Thanks Caren, for all you have done and meant to us and to the AP program. We are looking forward to working with you for many years to come."

How I Found the Golden Ratio on my CAS

By Lin McMullin

Our friend John Mahoney has an area problem that goes like this. Draw the line through the two points of inflection of a fourth-degree polynomial. The line intersects the graph of the polynomial in four places making three closed regions. From left to right the areas of these regions are in the ratio of 1:2:1. (See *The Mathematics Teacher* November 2002.) I was trying to develop an applet in Winplot to demonstrate this, but I never got it done. Something better came along. Here's what happened.

I knew if I started with the general form of a fourth-degree polynomial, $q(x) = m_4x^4 + m_3x^3 + m_2x^2 + m_1x + m_0$, that finding the x-coordinates of the points of inflection would be difficult enough, and then finding the other two points where the line through the points of inflection intersected the polynomial would be, for lack of a better word, ugly. (In fact the points of inflection are at $x = \dfrac{-3m_3 \pm \sqrt{9m_3^2 - 24m_4m_2}}{12m_4}$, and I didn't want to work with that and wouldn't have noticed the result reported here if I had. Sometimes being lazy is good!)

Instead I decided to make the points of inflection easy: $x = a$ and $x = b$, with $a < b$. Then the second derivative of $a < b$ looks like this: $q''(x) = 12m_4(x-1)(x-b)$.

Integrating twice with my TI V200's Computer Algebra System (CAS) I found that $q(x) = m_4x^4 - 2(a+b)m_4x^3 + 6abm_4x^2 + m_1x + m_0$

Then I wrote the equation of the line through the points of inflection $(a, q(a))$ and $(b, q(b))$ – on my CAS of course. The equation of the line is

$$y(x) = -(m_4a^3 - 3m_4a^2b - 3m_4ab^2 + m_4b^3 - m_1)x + (m_4a^3b - 3m_4a^2b^2 + m_4ab^3 + m_0)$$

One of the great things about using technology is that you don't have to write that down all you need to do is save it and recall it when you need it. Then I found the other

two points of intersection. I let the CAS solve the equation $q(x) = y(x)$. The intersection points are at $x = a$ and $x = b$ as expected, and also these two points:

$$x_L = \left(\frac{1+\sqrt{5}}{2}\right)a + \left(\frac{1-\sqrt{5}}{2}\right)b$$
$$x_R = \left(\frac{1+\sqrt{5}}{2}\right)b + \left(\frac{1-\sqrt{5}}{2}\right)a$$

When I saw this I stopped working on the original problem. The coefficients of the left intersection point, x_L, and the right intersection point, x_R involve the Golden Ratio, $\frac{1}{2}\left(1+\sqrt{5}\right)$, and its conjugate $\frac{1}{2}\left(1-\sqrt{5}\right)$. These numbers are the solution to the equation $x^2 = x+1$. I have no idea why this should be other than the fact that it is. I have no idea why the Golden Ratio shows up in *every* fourth-degree polynomial in this way, but it does.

There are various other relationships here that follow from those above. If $a < b$ then $x_L < a < b < x_R$. Then (1) $x_L + x_R = a + b$, (2) $\dfrac{b-a}{a-x_L} = \dfrac{b-a}{x_R - b} = \dfrac{1+\sqrt{5}}{2}$, (3) $x_R - x_L = \sqrt{5}\left(b-a\right)$, and (4) on the line containing the four points this same relationship exists between the lengths of the segments whose endpoints correspond to the numbers in (3). To prove this, draw a sketch and use some very elementary high school Geometry.

Finally, and this has absolutely nothing to do with the result above, the factored form of the sum and difference of two fifth powers also involves the Golden Ratio and its conjugate:

$$x^5 \pm a^5 = \left(x \pm a\right)\left(x^2 \mp \tfrac{1}{2}\left(1+\sqrt{5}\right)ax + a^2\right)\left(x^2 \mp \tfrac{1}{2}\left(1-\sqrt{5}\right)ax + a^2\right)$$

Cubic Exploration

1. Select a unique cubic with two critical points in the form
$f(x) = ax^3 + bx^2 + cx + d$. (a,b,c,d are all non-zero)
"Register" your cubic with the instructor to insure uniqueness.

2. Find the point of inflection of $f(x)$. Label it P.

3. Find the relative max (label it A) and the relative min (label it B) of $f(x)$.

4. Find the midpoint of $\overline{AB}$. What do you observe?

5. Write the equation L(x) for the horizontal line through the point of inflection.
Find the points where L(x) and $f(x)$ intersect.

6. L(x) and $f(x)$ bound two regions. Call these regions A_1 and A_2.
Find the area of A_1 and A_2. What do you observe?

7. Pick any point on $f(x)$ and label it R.
Find point S so that point P is the midpoint of $\overline{RS}$. (Use exact values.)
Is point S also on $f(x)$? What does this mean?

8. Draw an accurate sketch to illustrate your results.

9. Repeat for a second cubic. But this time we can design a cubic using "reverse engineering." First, pick the two x-values where you want the max and min to occur, $x = x_1$ and $x = x_2$.
Then, $f'(x) = (x - x_1)(x - x_2)$. Expand (FOIL) the expression for $f'(x)$.
Then find an antiderivative for $f'(x)$, i.e. get an expression for $f(x)$.
You now have values a,b, and c for the general cubic $f(x) = ax^3 + bx^2 + cx + d$.
Pick your own value for d.
Now your cubic is guaranteed to have max/min at "clean" x-values.

Parabola and Line Solution

Given A(1, 3) and B(6, 13) find the equations for a parabola and a line that pass through A and B and enclose area of 10.

$$\text{Slope of line} = \frac{13-3}{6-1} = 2$$
$$y-3 = 2(x-1) \Rightarrow y = 2x+1$$

$$\textit{Parabola}:$$
$$f(x) = ax^2 + bx + c$$
$$A(1,3):\quad f(1) = a\cdot 1^2 + b\cdot 1 + c = 3 \Rightarrow a+b+c = 3$$
$$B(6,13): f(6) = a\cdot 6^2 + b\cdot 6 + c = 13 \Rightarrow 36a+6b+c = 13$$

A third equation is needed.

$$\int_1^6 \left(ax^2 + bx + c - \left(2x+1\right)\right)dx = 10$$
$$\left(\frac{ax^3}{3} + \frac{bx^2}{2} + cx - x^2 - x\right)\Bigg|_1^6 = 10$$
$$\left(72a + 18b + 6c - 42\right) - \left(\frac{1}{3}a + \frac{1}{2}b + c - 2 = 10\right)$$
$$\frac{215}{3}a + \frac{35}{2}b + 5c = 50$$

Solving the system of three equations yields:

$$a = \frac{-12}{25}$$
$$b = \frac{134}{25}$$
$$c = \frac{-47}{25}$$

$$f(x) = \frac{-12}{25}x^2 + \frac{34}{25}x - \frac{47}{25}$$

Book Review by Deb Britt

Be Prepared for the AP Calculus Exam
by Mark Howell and Martha Montgomery
Contributions from Benita Albert, Tom Dick, Joe Milliet
Reviewed by Ray Cannon, Barbara Currier, Maria Letvin, Bernie Madison, Steve Olson, Nancy Stephenson and JT Sutcliffe

It should be obvious from the impressive list of authors, contributors and reviewers that this book would be good. It includes all those things that readers have been saying in workshops for years. It is all in one place, organized and includes a topic index.

Chapter 1 discusses the exam format, grading and tips. Chapters 2 through 9 discuss topics of the course content. These include Limits and Continuity, Derivatives, Applications of Derivatives, Integration, Applications of Integrals, Differential Equations, Parametric/Vector/Polar Functions, Series. The last two topics included in BC only. Chapter 10 contains annotated solutions to past free-response questions. Then you have an appendix with calculator skills. The best part for teachers would be the inclusion of 3 AB and 2 BC sample AP tests with answers and solutions.

Things I really liked

- the nice graphic on page 4 showing exam parts and times
- sample questions and solutions within each chapter
- Worksheets and solutions at the end of each chapter
- Clear labeling of BC only topics
- Practice exams and solutions with ideas for approach

Things I would have liked

- more discussion on Logistics Equations (after 2004 question)
- more discussion on Slopefields since this is newest topic
- to have this as a 3 hole punch notebook for easy reference
- more detailed use of TI-89
- placement of formulas in a single place for easy access

This is a "must have" book for every calculus teacher. You could easily do a summer workshop with this as the text. The book contains a companion website that contains annotated solutions to free response from past exams. I have known Mark Howell for a lot of years and he is one of the best.

Price is about $18. You can order **http://www.skylit.com**
Or email **sales@skylit.com**
ISBN 0-9727055-5-4

David Bressoud's article about the changing face of calculus can be found at the URLs for the two separate parts.

http://www.maa.org/features/faceofcalculus.html
http://www.maa.org/features/092404bressoud.html

2003 Multiple Choice by Topics

For those of you who have a copy of the 2003 AP Exam (See AP Central to order a copy)
I've attempted to align the 2003 AB multiple choice problems with the categories of topics in the Acorn Book. Here are two lists 1) by problem and 2) by category (some problems fall in more than one category), along with my answers.

John F. Mahoney, Benjamin Banneker Academic HS, Washington, DC

By Problem Number

1(E) computation of derivative
2(D) techniques of antidifferentiation
3(E) asymptotic behavior, derivative at a point
4(D) computation of derivative
5(D) techniques of antidifferentiation
6(C) limits of functions
7(B) derivative as a function, applications of derivative
8(B) techniques of antidifferentiation
9(A) computation of derivative
10(B) second derivative
11(C) techniques of antidifferentiation
12(E) concept of derivative
13(A) concept of derivative; continuity as a property
14(E) computation of derivative
15(D) derivative as a function
16(C) derivative at a point
17(A) computation of derivative; second derivative
18(A) derivative as a function
19(D) applications of antidifferentiation
20(D) concept of derivative, limits of functions
21(A) second derivative
22(D) Interpretation of definite integrals
23(E) Fundamental Theorem of Calculus, computation of derivative
24(C) derivative at a point
25(E) applications of derivative
26(B) computation of derivative
27(B) applications of derivative
28(E) second derivative
76(C) applications of derivative; calculator
77(C) applications of integrals; interpretation of definite integrals
78(C) applications of derivative
79(D) limits of functions
80(B) derivative as a function
81(D) applications of derivative; calculator
82(A) concept of derivative; applications of integrals
83(A) concept of derivative; calculator
84(A) applications of antidifferentiation; calculator
85(A) numerical approx. to integral
86(B) applications of integrals; calculator
87(B) second derivative; calculator
88(C) applications of integrals
89(D) computation of derivative; derivative at a point
90(B) derivative as a function; second derivative
91(E) applications of antidifferentiation; applications of derivative; calculator
92(D) Fundamental Theorem of Calculus; techniques of antidifferentiation; calculator

By Topic

applications of antidifferentiation 19(D) 84(A) 91(E)
applications of derivative 25(E) 27(B) 78(C) 91(E) 81(D) 76(C) 7(B)
applications of integrals 88(C) 86(B) 77(C) 82(A)
asymptotic behavior 3(E)
calculator 84(A) 91(E) 76(C) 81(D) 86(B) 83(A) 92(D) 87(B)
computation of derivative 1(E) 4(D) 9(A) 14(E) 26(B) 17(A) 89(D) 23(E)
concept of derivative 12(E) 20(D) 83(A) 82(A) 13(A)
continuity as a property 13(A) 79(D)
derivative as a function 7(B) 15(D) 18(A) 80(B) 90(B)
derivative at a point 89(D) 16(C) 24(C) 3(E) 79(D)
Fundamental Thm. of Calculus 23(E) 92(D)
interpretation of definite integrals 77 (C) 22(D)
limits of functions 20(D) 6(C)
numerical approx. to integral 85(A)
second derivative 10(B) 21(A) 28(E) 87(B) 17(A) 90(B)
techniques of antidifferentiation 92(D) 2(D) 5(D) 8(B) 11(C)

Excerpts from A Tour of Calculus by David Berlinski…

"As its campfires glow against the dark, every culture tells stories to itself about how the gods lit up the morning sky and set the wheel of being in to motion. The great scientific culture of the West – our culture – is no exception. The calculus is the story this world first told itself as it became the modern world."

"Space and time are the great imponderables of human experience, the continuum within which every life is lived and every river flows. In its largest, its most architectural aspect, the calculus is a great, even spectacular theory of space and time, a demonstration that in the real numbers there is an instrument adequate to their representation. If science begins in awe as the eye extends itself throughout the cold of space ……. Then in the calculus mankind has created an instrument commensurate with its capacity to wonder."

Audrey Weeks creator of Calculus In Motion & Algebra In Motion sent these excerpts
www.calculusinmotion.com

The 2003 Calculus AB and BC Released Exams are now available for purchase.
You can use the Order Form on AP Central to order your copy.
http://apcentral.collegeboard.com/article/0,3045,149-0-0-8020,00.html
AP Order Form (.pdf/504KB) It will soon be listed in the online College Board Store.
http://store.collegeboard.com
This publication contains a complete copy of the 2003 AP Calculus AB and BC Exams. For Section I, the multiple-choice section, you can see the questions and correct answers, along with statistical data on how students performed on each question. For Section II, the free-response portion of the exam, there are examples of students' actual responses, the scoring guidelines, and commentary that explains why the responses received the scores they did. There is background information on how the exam was developed, a description of the scoring process, and an overview of how the students performed on the exam. Also included is a diagnostic guide to help teachers analyze student results to find overall strengths and weaknesses in their understanding of AP Calculus. Finally, you can find out how scores are converted to AP grades and get advice on how to interpret those grades.
Packet of 10 2003 Calculus AB Exams
http://store.collegeboard.com/productdetail.do?Itemkey=725165
Packet of 10 2003 Calculus BC Exams
http://store.collegeboard.com/productdetail.do?Itemkey=725164

Susan Kornstein, K-12 Professional Development, The College Board

I was talking to an engineer over Christmas break and he said that the derivatives after jerk are snap, crackle, and pop. I'm assuming it's a more tongue in cheek idea but maybe someone can shed some more light on the origins. The website **http://www2.corepower.com:8080/~relfaq/jerk.html** links to a list of physical interpretations of third, fourth, fifth derivatives - including the vocabulary "jerk"

If you have a Smart Board® or access to the internet for your classroom, the following site may be a benefit.
http://astro.temple.edu/~dhill001/maxmin/maxmin.html

We Started by looking at solids and pottery. Then I got the idea of visiting a band instrument repair shop. They solid me clarinet bells for $5 a piece. The story follows in pictures you can see by following the links. It's fun!
http://www.mathteachtech.com/wathen/clarinet1.jpg
http://www.mathteachtech.com/wathen/clarinet2.jpg
http://www.mathteachtech.com/wathen/clarinet3.jpg
http://www.mathteachtech.com/wathen/clarinet4.jpg

Michael Wathen , R. A. Taft High School, Cincinnati, OH

For the new Calculus teachers, several years ago somebody put together a "Stuff you must know cold" for the Calc exams. I have that available for my students at our web site.
http://covenantchristian.org/bird/Smart/Calc1/Stuff%20you%20must%20know%20cold%20updated.pdf
If anyone is interested I've spent the time to make this document have blanks so they can quiz themselves. Also at the end of the year I have several similar quizzes that test them on how well they know this and simple questions associated with these formula. The quizzes are designed for AB but "Stuff You Must Know Cold" has it for both.

Sean Bird

Be sure to check out "2005: Calculus FRQ Instruction Commentary" on AP Central(also available as a PDF on the AP Calculus Course Home Pages)
http://apcentral.collegeboard.com/members/article/1,3046,151-165-0-8861,00.html
written by the AP Calculus Development Committee and Chief Reader, for valuable tips to help AP students and teachers understand how to interpret the free-response question instructions.
Here's a link right to David Lomen's article on AP Central
http://apcentral.collegeboard.com/members/article/1,3046,151-165-0-37292,00.html
The special focus of the theme materials included in College Board AP Calculus workshops this year is Differential Equations.

Susan Kornstein

DATES TO REMEMBER

February 26 - NCCTM Regionals in Raleigh & Hickory **www.learnnc.org/dpi/instserv.nsf/category7**
March 8 – AP Forum in Durham
March 11, 12 – Southeastern MAA in Raleigh
March 18 - T^3 Conference in Washington, DC
April 6 – 9 – NCTM in Anaheim, CA
May 3 – AP Calculus EXAMS

The Story of Polly Nomial

To prove once and for all that math can be fun, we present: Wherein it is related how that paragon of womanly virtue, young Polly Nomial (our heroine) is accosted by that notorious villain Curly Pi, and factored (oh horror!!!)

Once upon a time (1/t) pretty little Polly Nomial was strolling across a field of vectors when she came to the boundary of a singularly large matrix. Now Polly was convergent, and her mother had made it an absolute condition that she must never enter such an array without her brackets on. Polly, however, who had changed her variables that morning and was feeling particularly badly behaved, ignored this condition on the basis that it was insufficient and made her way in amongst the complex elements. Rows and columns closed in on her from all sides. Tangents approached her surface. She became tensor and tensor. Quite suddenly two branches of a hyperbola touched her at a single point. She oscillated violently, lost all sense of directrix, and went completely divergent. As she tripped over a square root that was protruding from the erf and plunged headlong down a steep gradient. When she rounded off once more, she found herself inverted, apparently alone, in a non-Euclidean space.

She was being watched, however. That smooth operator, Curly Pi, was lurking inner product. As his eyes devoured her curvilinear coordinates, a singular expression crossed his face. He wondered, "Was she still convergent?" He decided to integrate properly at once.

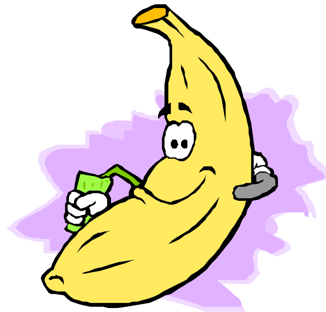
Hearing a common fraction behind her, Polly rotated and saw Curly Pi approaching with his power series extrapolated. She could see at once by his degenerate conic and dissipative that he was bent on no good. "Arcsinh," she gasped. "Ho, ho," he said, "What a symmetric little asymptote you have I can see you angles have lots of secs." "Oh sir," she protested, "keep away from me I haven't got my brackets on." "Calm yourself, my dear," said our suave operator, "your fears are purely imaginary." "I, I," she thought, "perhaps he's not normal but homologous." "What order are you?" the brute demanded. "Seventeen," replied Polly.

Curly leered "I suppose you've never been operated on." "Of course not," Polly replied quite properly, "I'm absolutely convergent." “Come, come," said Curly, "let's off to a decimal place I know and I'll take you to the limit." "Never," gasped Polly. "Abscissa," he swore, using the vilest oath he knew. His patience was gone. Coshing her over the coefficient with a log until she was powerless, Curly removed her discontinuities. He stared at her significant places, and began smoothing out her points of inflection. Poor Polly. The algorithmic method was now her only hope. She felt his digits tending to her asymptotic limit. Her convergence would soon be gone forever. There was no mercy, for Curly was a heavyside operator. Curly's radius squared itself; Polly's loci quivered. He integrated by parts. He integrated by partial fractions. After he cofactored, he performed runge - kutta on her. The complex beast even went all the way around and did a contour integration. What an indignity - to be multiply connected on her first integration. Curly went on operating until he completely satisfied her hypothesis, then he exponentiated and became completely orthogonal.

When Polly got home that night, her mother noticed that she was no longer piecewise continuous, but had been truncated in several places. But, it was too late to differentiate now. As the months went by, Polly's denominator increased monotonically. Finally she went to L'Hopital and generated a small but pathological function which left surds all over the place and drove Polly to deviation.

<u>The moral of our sad story is this:</u> "If you want to keep your expressions convergent, never allow them a single degree of freedom."

Author Unknown



Banana Split

In this exercise, you are going to find the volume of a banana. You will do this by using the technique of uniform cross-sections.

1. Select a banana.
2. Cut it in half lengthwise so that one half may be placed flat onto a piece of paper.
3. Place your half banana onto a piece of graph paper, oriented so that cross sections perpendicular to the x-axis will be roughly circular. Your work will be made easier if you use graph paper marked in the same units of length that you wish to use for calculating volume. E.g. Four squares to the inch means each line = 0.25 in.
4. Trace the outline of your banana on the graph paper. Remove the banana. Cover with ice cream and chocolate sauce and whipped cream … Oops! Lost myself there.
5. Consider the upper curve as Y1 and the lower curve as Y2. You need to find equations for Y1 and Y2.
6. Record at least eight data points for Y1 from the upper curve. Do likewise for Y2 from the lower curve.
7. Do a STAT Plot for each on your graphing calculator.
8. Get regression equations for Y1 and Y2. A Quartic regression will probably work best. Graph those on top of your plotted data.
9. Find the intersection points of Y1 and Y2. If they do not intersect, you will need to decide upon some reasonable left and right limits for integration.
10. <u>Set up a definite integral using uniform cross sections</u> to find the volume of your banana.

Try this exercise with other fruits or vegetables, such as a squash or zucchini. A cubic or even quartic regression might work best for those. If you want to have a check value to compare your calculated result, you could first immerse your banana into a full beaker of water and measure the overflow. If your vegetable has a linear axis of symmetry, the volume could also be found by treatment as a solid of revolution.

Function – First Derivative – Second Derivative Cards without Graphs

I did these functions/first derivatives/second derivatives on Sketchpad using a half a sheet of paper for each graph. I laminated them (without the equations/derivatives) and hung them around the room. The kids had to match them. I had hoped to use them for differential equations but haven't gotten around to thinking that through yet. If anyone wants the graphs to go along, they can email me at tmorris@greesboroday.org and I'll send them along. It's about 46 pages.

Trish Morris

Function #1: $f_1(x) = x^3 - 4x^2 + x + 2$

$$f_1'(x) = 3x^2 - 8x + 1 \qquad f_1''(x) = 6x - 8$$

Function #2: $f_2(x) = -2x^4 - 2x^3 + 7x^2 + 5x + 1$

$$f_2' = -8x^3 - 6x^2 + 14x + 5 \qquad f_2''(x) = -24x^2 - 12x + 14$$

Function #3: $f_3(x) = \frac{1}{4}x^2 + 2x - 3$

$$f_3'(x) = \frac{1}{2}x + 2 \qquad f_3''(x) = \frac{1}{2}$$

Function #4: $f(x) = -\frac{1}{3}x^3 + 2x^2 - 5x + 3$

$$f_4'(x) = -x^2 + 4x - 5 \qquad f_4''(x) = -2x + 4$$

Function #5: $f_5(x) = x^4 - 2x^3 + x^2 - 2x - 1$

$$f_5'(x) = 4x^3 - 6x^2 + 2x - 2 \qquad f_5''(x) = 12x^2 - 12x + 2$$

Function #6: $f_6(x) = -\frac{2}{3}x + 7$

$$f_6'(x) = -\frac{2}{3} \qquad f_6''(x) = 0$$

Function #7: $f_7(x) = 0.0025x^5 + 0.015x^4 - 0.13x^3 - 0.54x^2 + 1.44x$

$$f_7'(x) = 0.0125x^4 + 0.060x^3 - 0.39x^2 - 1.08x + 1.44$$

$$f_7''(x) = 0.0500x^3 + 0.18x^2 - 0.78x - 1.08$$

Function #8: $f_8(x) = -\frac{1}{3}x^2 + \frac{1}{2}x + 9$

$$f_8'(x) = -\frac{2}{3}x + \frac{1}{2} \qquad f_8''(x) = -\frac{2}{3}$$

Function #9: $f_9(x) = \frac{44}{5}x - 2$

$$f_9'(x) = \frac{44}{5} \qquad f_9''(x) = 0$$

Functions #10:

$$f_{10}(x) = 0.001x^5 + 0.008x^4 - 0.043x^3 - 0.302x^2 + 0.156x + 1.080$$

$$f_{10}'(x) = 0.005x^4 + 0.032x^3 - 0.129x^2 - 0.604x + 0.156$$

$$f_{10}''(x) = 0.02x^3 + 0.096x^2 - 0.258x - 0.604$$

Function #11: $f_{11}(x) = 0.125x^3 - 0.25x^2 - 2x + 4$

$$f_{11}'(x) = 0.375x^2 - 0.5x - 2 \qquad f_{11}''(x) = 0.75x - 0.5$$

Function #12: $f_{12}(x) = 0.04x^4 + 0.24x^3 - 0.60x^2 - 1.76x + 3.36$

$$f_{12}'(x) = 0.16x^3 + 0.72x^2 - 1.20x - 1.76$$

$$f_{12}''(x) = 0.48x^2 + 1.44x - 1.20$$

Function #13: $f_{13}(x) = 5\sin x$

$$f_{13}'(x) = 5\cos x \qquad f_{13}''(x) = -5\sin x$$

Function #14: $f_{14}(x) = -8\cos\left(\frac{1}{2}x\right)$

$$f_{14}'(x) = 4\sin\left(\frac{1}{2}x\right) \qquad f_{14}''(x) = 2\cos\left(\frac{1}{2}x\right)$$

Function #15: $f_{15}(x) = 4 + \frac{1}{2}\tan x$

$$f_{15}'(x) = \frac{1}{8}\sec^2\frac{x}{4} \qquad f_{15}''(x) = \frac{1}{16}\sec^2\frac{x}{4}\tan\frac{x}{4}$$

Function #16: $f_{16}(x) = -x - 3\cos x$

$$f_{16}{}'(x) = -1 + 3\sin x \qquad f_{16}{}''(x) = 3\cos x$$

Function #17: $f_{17}(x) = \dfrac{1}{x}$

$$f_{17}{}'(x) = -\frac{1}{x^2} \qquad f_{17}{}''(x) = \frac{2}{x^3}$$

Function #18: $f_{18}(x) = \dfrac{3x}{x+1}$

$$f_{18}{}'(x) = \frac{3}{\left(x+1\right)^2} \qquad f_{18}{}''(x) = \frac{-6}{\left(x+1\right)^3}$$

Function #19: $f_{19}(x) = \dfrac{x^2 - 6x + 18}{x - 3}$

$$f_{19}{}'(x) = \frac{x^2 - 6x}{\left(x-3\right)^2} \qquad f_{19}{}''(x) = \frac{18}{\left(x-3\right)^3}$$

Function #20: $f_{20}(x) = -\dfrac{20x}{x^2 + 2}$

$$f_{20}{}'(x) = \frac{20x^2 - 40}{\left(x^2+2\right)^2} \qquad f_{20}{}''(x) = \frac{40\left(6 - x^2\right)}{\left(x^2+2\right)^3}$$

Function #21: $f_{21}(x) = 2e^x$

$$f_{21}{}'(x) = 2e^x \qquad f_{21}{}''(x) = 2e^x$$

Function #22: $f_{22}(x) = xe^{x/2}$

$$f_{22}{}'(x) = e^{x/2} + \frac{x}{2}e^{x/2} \qquad f_{22}{}''(x) = e^{x/2} + \frac{x}{4}e^{x/2}$$

Function #23: $f_{23}(x) = e^{-3x}$

$$f_{23}{}'(x) = -3e^{-3x} \qquad f_{23}{}''(x) = 9e^{-3x}$$

Function #24: $f_{24}(x) = 5^{2x} + 2x$

$$f_{24}{}'(x) = 2 \cdot 5^{2x} \cdot \ln 5 + 2 \qquad f_{24}{}''(x) = 4 \cdot 5^{2x}\left(\ln 2\right)^2$$

Function #25: $f_{25}(x) = \sin x\left(e^{-0.1x}\right)$

$$f_{25}'(x) = \cos x\left(e^{-0.1x}\right) - 0.1\sin x\left(e^{-0.1x}\right)$$

$$f_{25}''(x) = -\sin x\left(e^{-0.1x}\right) - 0.2\cos x\left(e^{-0.1x}\right) + 0.05\sin x\left(e^{-0.1x}\right)$$

Function #26: $f_{26}(x) = -\ln\left|x\right|$

$$f_{26}'(x) = -\frac{1}{x} \qquad f_{26}''(x) = \frac{1}{x^2}$$

Function #27: $f_{27}(x) = x\ln(x+9)$

$$f_{27}'(x) = \ln\left(x+9\right) + \frac{x}{x+9} \qquad f_{27}''(x) = \frac{1}{x+9} + \frac{9}{\left(x+9\right)^2}$$

Function #28: $f_{28}(x) = \cos x \cdot \ln\left|x\right|$

$f_{28}'(x) = -\sin x \cdot \ln\left|x\right| + \dfrac{\cos x}{x}$ for $x > 0$ and

$f_{28}'(x) = -\sin x \cdot \ln\left|x\right| - \dfrac{\cos x}{x}$ for $x < 0$

$f_{28}''(x) = -\cos x \cdot \ln\left|x\right| - \dfrac{\sin x}{x} - \left(\dfrac{x\sin x + \cos x}{x^2}\right)$ when $x > 0$ and

$f_{28}''(x) = -\cos x \cdot \ln\left|x\right| - \dfrac{\sin x}{x} + \left(\dfrac{x\sin x + \cos x}{x^2}\right)$ when $x < 0$

Function #29: $f_{29}(x) = \ln\left|x\right| - 3x$

$f_{29}'(x) = \dfrac{1}{x} - 3$ when $x > 0$ and $f_{29}'(x) = -\dfrac{1}{x} - 3$ when $x < 0$

$f_{29}''(x) = -\dfrac{1}{x^2}$ when $x > 0$ and $f_{29}''(x) = \dfrac{1}{x^2}$ when $x < 0$

Function #30: $f_{30}(x) = 6^x + \cos x$

$$f_{30}'(x) = 6^x \ln 6 - \sin x \qquad f_{30}''(x) = 6^x\left(\ln 6\right)^2 - \cos x$$

Common Tangent Problem #1

The x-axis is tangent to the graph of both $f(x) = x^2$ and $g(x) = x^3$
Find another line (exact form) which is tangent to both graphs.

Common Tangent Problem #2

Let $f(x) = x^2 - 3$ and $g(x) = -x^2 + 6x - 8$

(1) Sketch the graphs of $f(x)$ and $g(x)$ on the same axes.

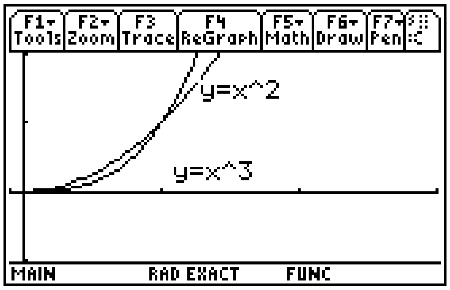
(2) Sketch the graph of $T(x) = 2x - 4$ on the same axes.

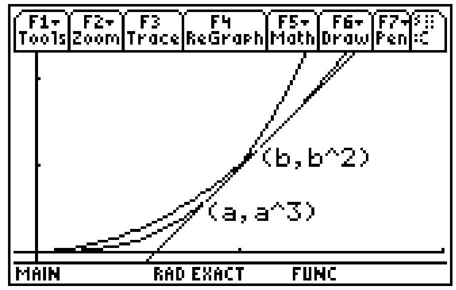
(3) Find any points of intersection among the three graphs.

(4) What is the slope of each of the curves at these points of intersection?

(5) There is another line that has the same relationship to $f(x)$ and $g(x)$ as $T(x)$.
Find the equation for this line.
Show each step of your solution.

Solution to Common Tangents #1





Label the point of tangency on $f(x) = x^2$ as B(b,b^2).

Label the point of tangency on $g(x) = x^3$ as A(a, a^3).

$f'(x) = 2x$ and $g'(x) = 3x^2$

$f'(b) = 2b$ and $g'(a) = 3a^2$

$3a^2 = 2b \quad \Rightarrow \quad b = \dfrac{3a^2}{2}$

Slope of AB by slope formula $= \dfrac{b^2 - a^3}{b - a}$

Now: $\dfrac{b^2 - a^3}{b - a} = 3a^2 = 2b$

$$\frac{b^2 - a^3}{b - a} = 3a^2$$

Substitute $b = \dfrac{3a^2}{2}$

$$\frac{\left[\frac{3a^2}{2}\right]^2 - a^3}{\frac{3a^2}{2} - a} = 3a^2$$

$$\frac{\left[\frac{9a^4}{4}\right] - a^3}{\frac{3a^2}{2} - a} = 3a^2$$

Simplify fraction on left

(multiply by LCD of 4)

$$\frac{9\text{a}^4 - 4a^3}{6a^2 - 4a} = 3a^2$$

$$9a^4 - 4a^3 = 3a^2(6a^2 - 4a)$$

$$9a^4 - 4a^3 = 18a^4 - 12a^3$$

$$9a^4 - 8a^3 = 0$$

$$a^3(9a - 8) = 0$$

$$a = 0 \;\text{ or }\; a = \frac{8}{9}$$

$$\therefore \quad b = \frac{3\left(\frac{8}{9}\right)^2}{2} = \frac{32}{27}$$

Equation of line:

$y = mx + b_1$ using $\text{B}(b, b^2) = (\frac{32}{27}, \frac{1024}{729})$

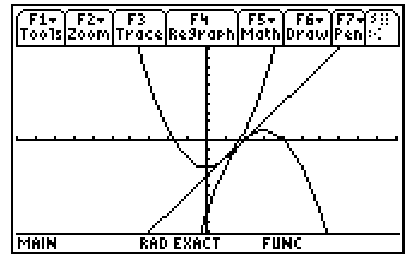
$$m = 2b = \frac{64}{27}$$

$$\frac{1024}{729} = \frac{64}{27} \cdot \frac{32}{27} + b_1$$

$$b_1 = \frac{-1024}{729}$$

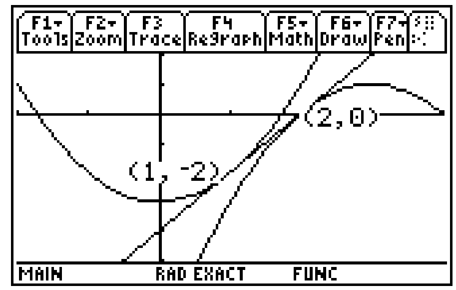
Equation is now: $y = \frac{64}{27}x - \frac{1024}{729}$

Solution to Common Tangents #2 Problem



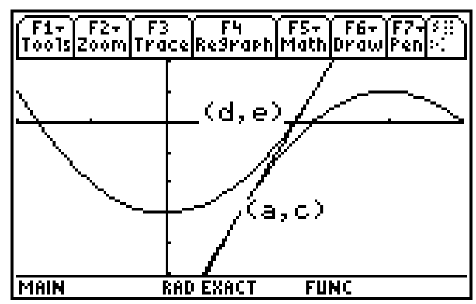
$f(x) = x^2 - 3$
$g(x) = -x^2 + 6x - 8$
$T(x) = 2x - 4$

Intersection Points:
$f(x)$ and $\mathrm{T}(x)$:
$x^2 - 3 = x^2 + 6x - 8 \Rightarrow x = 1 \Rightarrow (1,-2)$



$g(x)$ and $T(x)$:
$-x^2 + 6x - 8 = 2x - 4 \Rightarrow x = 2 \Rightarrow (2,0)$

Let other common tangent be P(x),
with intersection points (a,c) and (d,e)



$P(x) = mx + b$

When $x = d$: When $x = a$:

$d^2 - 3 = m \cdot d + b$ $\quad - a^2 + 6x - 8 = m \cdot a + b$

Eliminating b yields:
$d^2 - 3 - md = -a^2 + 6a - 8 - ma$

$f'(d) = 2d$ $\quad g'(a) = -2a + 6$
$P'(d) = m$ $\quad P'(a) = m$
$\Downarrow$ $\quad \Downarrow$
$m = 2d$ and $m = 3 - a$

Substituting and eliminating variables yields:
$2a^2 - 6a + 4 = 0 \Rightarrow a = 2$ and $a = 1$
Discarding $a = 2$ as already found on $g(x)$,
and using $a = 1$: $g(1) = -3 \Rightarrow (1,-3)$
Now $d = 3 - a = 2$
$f(2) = 2^2 - 3 = 1 \Rightarrow (2,1)$

Line through (2, 1) and (1, -3): $y = 4x - 7$