The North Carolina Association Of Advanced Placement® Mathematics Teachers Newsletter

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Web Address
www.ncaapmt.org/calculus

NOTE: Any NC member wishing to be considered for nomination to the Board should contact one of the present board members listed above. Contact Martha Ray at raym@gcsnc.com We presently have openings in the eastern and western regions.

Notes from the President's Desk

Common Core State Standards (CCSS) is presently a major focus for educators. A "Common Core State Standards Alignment" was developed by the College Board to document the existing correspondence between CCSS and AP Courses. AP Calculus AB, AP Calculus BC, AP Statistics, and AP Computer Science A as well as the AP English courses were included in this study. The AP Mathematics course alignment can be found in Appendix B of the following link: http://professionals.collegeboard.com/profdownload/pdf/RR2011-8.pdf. The alignment chart for mathematics begins with grade 6 and extends through high school mathematics. AP course materials were used to develop this alignment. An "X" is used to indicate when AP course(s) correlates with the CCSS. Standards identified with (+) are considered fourth mathematics concepts.

High School Functions is one conceptual category I would like to mention. Interpreting Functions and Building Functions are two of the domains of Functions. F.IF.9 (Functions – Interpreting Functions-Standard 9) is "Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions). This standard emphasizes a multi-representational approach and will be taught in Algebra 1 through AP Calculus.

As I read, study, "unwrap" the CCSS I find myself thinking, "If students are proficient in these standards, they are going to do very well in the AP courses." Mathematics Vertical Teams have become more important than ever as we begin with the CCSS in fall 2012. Appendix B will be instrumental in making the connections across the grade levels in preparing our students for college and career.

Martha Y. Ray, NCAAPMT President
Guilford County Schools
Secondary Mathematics Curriculum Specialist

Notes From The Secretary's Desk

The reading was especially exciting for me this year. Since I missed my Ph.D. graduation at Ohio University to attend the reading, Mike Boardman and Steve Kokoska dropped by my reading "pod" to "hood" me. Tom Dick walked me down the aisle of friends gathered on either side, while Mark Howell, Guy Mauldin and Judy Broadwin led the humming of *Pomp and Circumstance*. I will not try to mention all the names of those in attendance for fear I will leave out some. I only mention Mark, Guy and Judy because they have been through so many of the readings with me. It was great having Judy Broadwin back. Another exciting surprise was having Ray Cannon, former Chief Reader drop in for a night to visit and sign the new Rogawski book he helped co-author (details of this book are later in the newsletter). The clickers we used to answer survey questions were a novelty piece that made things more interesting and I am sure helped the presenters gather useful data. It felt a little more assembly line for me - with only 3 questions in the AB rooms, there were two very long days on the same question with no change of problems. Having read the Alternate exam and the Form B for the past two years, I had forgotten what those long days feel like. My table partner was an acorn (new reader) and was a lot of fun (his comments can be read in the New Reader article).

My one-year part-time contract with Buncombe County Schools expired June 13. So, it feels great to be overeducated with a new Ph.D. and unemployed! I am doing contract work with The College Board and ETS. If anyone knows of a good place - call me. I am deeply concerned that our honors mathematics courses are losing some of their rigor - seems to be from student placement. This will have implications for AP Calculus teachers.

If anyone is interested in taking over for me as editor of this newsletter, please let me know. I think it is time for new ideas and for someone who might like to design a new format.

Deb Britt, Mars Hill, NC, dgb531@aol.com

Please remember to **renew your membership** to receive the two yearly newsletters. You can send your $5.00 check, payable to NCA^{2}PMT, to Jeff Lucia, 718 Landsdowne Road, Charlotte, NC 28270. Email address is jeff.lucia@providenceday.org.

JOIN US ON FACEBOOK !!!

The NCAAPMT now has an official Facebook group that members may use to network and post announcements. When you are logged into Facebook, just do a search for NCAAPMT and ask to be admitted to the group.

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2011 AP Calculus Reading Information

2011 AP Calculus Reading Opening Session

Michael Boardman welcomed us back to Kansas City and within a few hours, things were up and running. The major new change was that during the opening Session and question briefings, we used "Clickers" to answer survey questions and record whether we were grading the samples correctly. This immediate feedback assisted the presenters with decisions regarding how well the readers understood the scoring.

There were 342,593 total calculus examinations (256,950 AB and 85,643 BC) for 2011. The readers graded 2,055,558 questions with a possibility of awarding 18,500,022 points. Chief Reader Michael Boardman pointed out that if he were to grade these exams alone, it would take 21 years, 76 days, 4 hours, 57 minutes. There were 859 participants at the reading, including the Chief Reader and the new Chief Reader Designate (Steve Kokoska). The CR and the CR-D do not score papers so 857 readers graded papers. This year, there were 132 new readers. 92 of the readers were Table Leaders. Readers represented all 50 states and several territories and countries. 52% of the graders were high school and 47% were college. There were 3 rooms that graded the alternate examinations and two rooms who scored the Form B exams.

It should be noted that in the final debriefing, there were common concerns mentioned for all questions. Those concerns involved students missing points due to their lack of communicating mathematics. Often students failed to label expressions or show work indicating what they were thinking in their solution. In almost all cases, students with only answers received no credit in all question parts - even if the answers were correct. Parentheses errors often cost points. There were not generally as many degree mode rather than radian mode errors as in the past. Students are not using table values provided or are using the wrong table values. Students have confusion over average value of a function, average rate or the word average. Students often use a string of equal signs between expressions where the pieces are not equal. It was astonishing how many students made errors in simple subtraction, addition, multiplication, or division of numbers - especially when they could have received points by merely setting up the expression and stopping. Students often are not precise in relating their knowledge of the hypothesis and conclusion of a theorem. Some think setting derivatives equal to zero is all the work needed or they set derivatives equal to zero and solve without explicitly stating they are doing that. Students often do not understand what "in terms of k" (or some other variable) means - giving numerical answers instead. Fundamental Theorem of Calculus issues frequently arose - students often mishandled the initial condition. It was clear that students often feel uncomfortable with letters other than x and some even wrote that they had this difficulty. It was surprising the number of students who do not know how to solve a separable differentiable equation, and once again, this costs them 5 points.

Unofficial 2011 Means

Note: Almost all exams had been graded when this data was computed but these results are not official.

Questions	All	Without 0s & dashes	Questions	All	Without 0s & dashes
AB 1	2.79	3.54	BC 1	5.24	5.53
AB 2*	3.29	4.30	**BC 2***	5.50	5.87
AB 3	4.64	5.14	BC 3	5.50	6.37
AB 4*	2.42	3.44	**BC 4***	4.14	4.48
AB 5*	1.63	3.57	**BC 5***	3.53	4.82
AB 6	3.03	3.72	BC 6	3.97	4.58

*****indicates common question on AB and BC Exams

Unofficial 2011 Score Distributions

From Trevor Packer's twitter:
Calculus **BC** scores, 2011: 47.0% = 5; 16.3% = 4; 17.2% = 3; 5.9% = 2; 13.6% = 1. (These may shift slightly as late exams arrive.)

AP Calculus **AB** scores, 2011: 21.0% = 5; 16.3% = 4; 18.5% = 3; 10.8% = 2; 33.4% = 1. (These may shift slightly as late exams arrive.)

The Changing of the Chief Reader
Stephen Davis - Davidson College, Davidson, NC

This July was Michael Boardman's 18th AP Calculus Reading, dating to 1994 when he came as an "acorn" from Pacific University in Forest Grove, Oregon. Michael has been a Reader, Table Leader, Question Leader, Exam Leader, and has served as the Chief Reader the past four years. He also was the first moderator for the EDG (Electronic Discussion Group). He joined a select group of those Chief Readers to preside over a grading that included his own daughter's exam. Michael brought intensity to the role of Chief Reader that was driven by his concern that students be assessed fairly and accurately. A key point of emphasis for him was that the Free Response section of the exam is the place where students can communicate their understanding of calculus. Part of this implies an obligation for the student to let us in on how their solutions are developed; the Multiple Choice section is the place for reporting just answers.

Michael is succeeded as Chief Reader by Stephen Kokoska of Bloomsburg University in Pennsylvania. This July was Steve's 22nd AP Calculus Reading, and like Michael, he has been in all positions, Reader, Table Leader, Question Leader, and Exam Leader, in preparation for the position. We look forward to Steve's leadership over the next four years.

College Board Open Forum on AP Calculus
Arthur Rosenthal - Salem State University, Salem, MA

Trevor Packer, the College Board's Vice President responsible for leadership of the Advanced Placement program, led a College Board Open Forum on the AP Calculus program on June 10th during the AP Calculus Reading held in Kansas City, Missouri. Most of the Open Forum was devoted to Trevor answers and comments on questions asked by teachers and professors. Trevor mentioned that he had taken an AP course in Calculus when he was a high school student, but had not been confident enough in his performance in that course to make a serious effort at his AP Calculus exam. Nevertheless, he thought his AP Calculus course really helped him achieve a top score out of 200 students in his university calculus course.

Trevor announced one change to the AP Calculus exam involving changing all the multiple choice and open response questions each year in the version of the exams administered outside of North America. With the exam for international students being changed each year, the multiple choice questions used could be released every year (but only for the "Form B" version used for international students), instead of only on the five-year cycle as is currently the case.

The change implemented this year involving no deductions for incorrect answers in the multiple-choice section was discussed. Another change implemented this year involving the modification of the free response section to include two problems with a graphing calculator required and four problems with no calculator allowed (from the three – three split used before 2011) was also discussed. The current intention is to keep these changes in effect for the 2012 AP Calculus exams, with free response questions to be scored from June 10 – 16, 2012 in Kansas City, Missouri.

The issue of over 30% of students receiving a score of "1" on the Calculus AB exam was raised. It seems that many high schools are being pressured to having more students take the AB exam than have mastered the prerequisites for Calculus. To distinguish which of those students make a reasonable attempt to answer the questions, some people asked whether a score of "0" could be given to students who predominantly give no answers or answers with no mathematical content to most questions. Trevor Packer responded that this change is being seriously considered, but a disadvantage of it might be that students could easily remove the possibility of getting a "0" by writing something mathematically related for each question, if they knew they or their school would be adversely affected by leaving their answers to questions blank.

The issue was raised about whether the prohibition of the use by students of mechanical pencils could be changed. Trevor Packer said this would be considered.

Trevor was asked what he thought about the policies being instituted by some states involving evaluating and rewarding teachers based on the scores their students obtain on AP exams. Trevor responded that he thought that policy is "ludicrous" and that the College Board is being resistant to changes to its policies being requested by some states to facilitate that method of teacher evaluation.

Some teachers requested that the College Board provide them the numerical scores their students obtain (both the total scores and the scores on the various subtypes of questions) instead of only the scores using the categories from 1 to 5. They felt that it would be helpful, for example, to know that a student who received a "5" actually got a 65% numerical score on their AP exam. Some professors also feel that numerical scores would help universities know the fraction of the Calculus material that has been mastered by incoming students who took advanced placement exams. Trevor said these changes could be considered, but they had advantages and disadvantages.

Updates from Trevor Packer on these issues and other issues may be found by going to http://www.twitter.com and searching for @AP_Trevor .

Test Development Committee Night
Mark Howell - Gonzaga High School, Washington, DC

At the 2011 AP Calculus Reading in Kansas City, the Test Development Committee appeared at the "Meet the Committee" night on June 11. Committee co-chair Stephen Davis from Davidson College held court, accompanied by all of the committee members: Robert Arrigo from Scarsdale High School in New York, Tom Becvar, newly-appointed committee co-chair from St Louis University High School in Missouri, Vicki Carter from West Florence High School in South Carolina, Kathleen Goto from Iolani School in Hawaii, Donald King from Northeastern University in Massachusetts, and Tara Smith from the University of Cincinnati in Ohio.

Davis described the committee's responsibilities and meeting schedule. Their workload is daunting, as there are now 4 exams every year: the "operational" exam and its "alternate" companion for North and South America, including Alaska and Hawaii, and the international exam and its alternate companion. The alternate exams are administered at a later day to students who have a valid approved reason for not taking the exam on the first scheduled date. Since there are three common problems on each pair of AB and BC exams, the committee will need to work with a total of 36 questions annually. They work together 3 times a year face-to-face and hold several additional virtual meetings.

Davis addressed head-on a couple of challenges faced by the committee. In particular, he pointed to issues relating to students' use of Computer Algebra Systems (CAS) on the free response part of the2011 exam. Davis emphasized that the committee is driven by equity considerations on both sides: that students who use either a CAS-equipped or non CAS-equipped calculator are treated fairly by exam questions as well as by scoring rubrics.

The committee responded to a few questions after the presentation. On the topic of series, Kathleen Goto reported her intention to try to begin covering the topic earlier in her course next year. Some teachers have difficulty at the end of the course finding time for adequate coverage. A questioner pointed out the heavy emphasis on average value of a function on the 2011 free response. The committee responded that those sorts of peculiarities are not planned out deliberately, but occur more as a consequence of the complex task of assembling so many exams that need to cover a variety of specifications for function type, function representation, and topics.

There are no other major changes on the horizon for AP Calculus. Sometime in the near future, a complete AP Calculus exam will be released every year (a fact that will be welcomed by school teachers). [Earlier in the week, at the College Board Open Forum, a Board representative announced the report to the teacher sent to each school in the fall will be available electronically and have much more detail in it for the teacher. The timing of this change is uncertain.]

Professional Night #1: Technology Night
Marti Freihofer - St. Henry District High School, Erlanger, KY

The goal of technology night was to highlight the uses of technology in calculus instruction. This was accomplished through five short, fifteen minute presentations given by Ben Klein, Doug Meade, Mark Howell, Ruth Dover, and Tom Dick. The evening was quite informative and was followed by a hands-on exploration session for the second professional night.

The first presentation of the evening was given by Ben Klein, demonstrating uses of the Casio ClassPad CAS calculator. CAS refers to computer algebra software. He showed how the Casio could be used to find area both algebraically and graphically, graph equations, solve for intercepts, solve equations for the variable *x*, evaluate limits, and create spreadsheets to calculate Riemann Sums. The Casio has drop-down menus similar to certain Texas Instruments calculators (the one I'm most familiar with is the TI-Voyage 200), and the keyboard has templates built into it for most math operations. The one major advantage that I saw was that the Casio allows you to see both the graph and the algebra that you are performing at the same time without switching between screens.

The second presentation of the evening was given by Mark Howell, and it dealt with the HP40gs or "CAS with Gas" as he termed it. This is a graphing calculator that is the upgraded version of the HP39gs (CAS was added). The representations of functions that are dealt with in calculus (symbolic, graphical, and numeric) are more explicit on this calculator than on others that I am familiar with. The keyboard is simple and sensible, and most students or teachers would become familiar with it in short order. One major difference is that the HP CAS is a 2D editing environment, and it allows you to highlight an expression that you are working with in the CAS and send it to the graph (you don't have to retype it). There is also a "step-by-step" mode that shows each step involved in the calculations you are performing. This is a great way to use CAS to demonstrate some of the rules for differentiation and integration in our classrooms.

Doug Meade from the University of South Carolina had the third presentation for the night, and it covered some aspects of GeoGebra software. This is a free GNU ed software package that is available for download at www.geogebra.org and is mainly used for mathematics visualization. The program may be run as a stand-alone program or through a browser. There are several resources available to help you get started, such as GeoGebra Quickstart, GeoGebra Book, and GeoGebra Channel (which is on YouTube). Other resources have been created by Father Mike May from St. Louis University, and can be found on his websites:

http://prep11geogebra.pbworks.com/w/page/37663288/CalculusOverview
www.slu.edu/classes/maymk/GeoGebra

Another useful website is http://webspace.ship.edu/msrenault/GeoGebra.html which is maintained by Marc Renault.

Ruth Dover gave her presentation on Mathematica next. This is a text-based program that allows the user to plot functions, evaluate derivatives and antiderivatives, and create animations. While I found the animations to be very nice and believe that they would be useful in helping calculus students to visualize what they are learning, I found the program to be rather complicated with many symbols and commands to learn. If you are familiar with computer programming, you might enjoy using this program. There is a demo site at http://demonstrations.wolfram.com which is free and will allow you to use the demonstrations without downloading Mathematica (the program itself is available through www.wolfram.com).

The final presentation of the night was given by Tom Dick on the TI-Nspire CAS. He demonstrated the use of this calculator for viewing differential equations, 3-D graphs (these can be graphed and manipulated), and several illustrations (including one that involved a picture of an urn that corresponded to a graph of the height of liquid dependent on the amount of liquid poured into the urn). The equation writer embedded in this calculator is similar to that in Microsoft Word. Tom indicated that some illustrations are available for free on the web and suggested that you needed to do a Google search. A nice feature of the TI-Nspire CAS is that the CAS is recognized by spreadsheets which will allow you to explore Taylor polynomials without doing all of the calculations by hand.

All of the presentations were well thought out and shared some very useful technological applications for the calculus classroom. There is a wealth of information available on the web for each of these calculators and software programs, and I encourage you to explore each of them to find what will work best in your classroom.

Note: Professional Night #2 was a hands-on session allowing readers to tryout any of the technology discussed by the five presenters at Professional Night #1.

Grading the Alternate Exams
Larry Peterson - Northridge High School, Layton, UT

Each year for a variety of reasons, some students are unable to take the AP Calculus exam on the designated date. For students who have a legitimate reason for missing the test, the College Board offers an alternate exam called Form A for both AB and BC students. This year, there were approximately 5000 AB and 4000 BC students who took an alternate form.

These exams are similar in content and difficulty to the Operational and Form B exams. However, due to the difficulty in determining the cutoff scores for a test with a small number of students, some of the questions may be recycled in future years. Consequently, the test is kept secure and is not released to the general public.

The grading for these exams was done in Kansas City, MO at the same time as the Operational exam. Forty experienced readers who have demonstrated an ability to learn and apply the standards quickly were formed into two separate reading groups called tables. They were required to sign a nondisclosure agreement stating they would not reveal the specific content of the questions.

The readers were trained on the first operational exam question and graded it for a few hours along with all the other readers. They were then moved to a secure room where they were briefed on the Form A Exam by Julie Clark and Stephen Davis. For each question, readers were given a rubric and discussion about special cases. Student samples were also used to check the readers' understanding of the rubric. Then they graded questions in pairs. They worked with each other and their Table Leaders to make sure they fully understood the grading scale. Table Leaders reviewed their work at regular intervals to make sure they were on track with the standards. The same care was taken to insure that students were given every opportunity to be judged fairly on their work. As with the regular exams, no reader graded a student's work more than once.

Over the course of the next four days, these readers would cycle in and out of the training and grading flow of the main exam. Each of the Alternate Exam readers was trained on three regular exam questions and four or five Alternate Exam questions. There are also provisions made for BC students to receive an AB sub-score just like the operational exam.

Some students may feel the Alternate Exam is more challenging that the regular exam or that the questions are not graded as carefully. This is certainly not the case. Students and teachers can be assured that they are treated just as fairly as those students who took the operational exam.

Professional Speaker Night
Trish Morris - Greensboro Day School, Greensboro, NC

On Tuesday, June 14 current NCTM president J. Michael Shaughnessy addressed the AP Calculus readers about "eliminating eternal algebra." To begin his address, "AP Calculus: Too Much of a Good Thing," Mr. Shaughnessy read and showed the audience several articles from various newspapers ranging from the New York Times to the Oregorian that dealt with math in the news. The commentaries used numerical descriptions of topics that ran the gamut of public interest as well as the obvious items in the sports and financial pages. To make sense of news and information we receive every day, knowledge of data analysis and statistics is much more helpful to the average consumer than the understanding of calculus. His argument is that math educators are not necessarily preparing future citizens to be discriminating interpreters of data that is presented to them. He states that the "algebra – calculus" gateway is a mathematical abyss for many students.

As a high school student trained under the SMSG program of the sixties and seventies, Mr. Shaughnessy didn't have calculus in high school. He described his fourth year math course as "eclectic." Now calculus is the main driver for students to transition from high school mathematics to college. Clearly more students are taking it in high school and even more are being funneled into "the rush to calculus." MAA as well as NCTM share this concern. David Bressoud, current MAA president, indicated that out of 160,000 students that scored three or better on the 2005 AB exam, only 104,000 were registered in a Calculus II course. In the fall of 2010 MAA surveyed 14,000 college students and about 700 instructors. Of those students surveyed, 68% of them had calculus in high school (56% in AB and 12% in BC). Sixty-one percent of the students who had had calculus in high school earned an A in their high school calculus course. However, only 33% of those same students who took the AB or BC exam achieved a grade of 3 or higher on it. For preliminary findings and more accurate statistics see the website listed here: http://www.maa.org/columns/launchings/launchings_05_11.html.

Mr. Shaughnessy's point in relating these statistics to us was to help persuade us that "Endless Algebra is an endless pathway that is deadly for a lot of students." In both high school and college, mathematics education is not serving students well. He feels math education needs to place more emphasis on modeling, dealing with variability, and decision making under uncertainty. Math education during the last two years of high school and the first two years of college must address the need for mathematical proficiency and the understanding necessary to be successful consumers and citizens in the twenty-first century.

In the question and answer period that followed, Mr. Shaughnessy acknowledged and agreed with most of the points that were raised by the audience:

- High school calculus is the most appropriate path for our gifted and talented math students. However, that does not mean they should not have exposure to the types of courses that Mr. Shaughnessy is referring to.
- Does the content and concepts that he is advocating fit the Common Core Standards which have been adopted by forty-four states?
- Where is this curriculum going to come from? He mentioned the work going on at the Dana Center at the

University of Texas as well as the Modspar program at Ohio State. Modspar (modeling and spatial reasoning) focuses on discrete and continuous modeling, geometric modeling, and spatial reasoning.

- Teacher training for such an addition needs to be addressed.
- Probably the greatest concern voiced by the audience was the impetus from college admission offices that expect applicants to take the most advanced math course available in high school and not the one that would probably benefit them the most.

Mr. Shaughnessy raised some very valid points regarding the majority of high school math students. After grading some disappointing answers to questions, it is apparent that not every student in high school calculus classes is prepared to take such a course or is receiving the proper training. Such a student would benefit from an alternative to calculus.

New Reader (Life as a Potential – Future – Mighty Oak)
Mark Fischer - Wilbraham & Monson Academy, Wilbraham, MA

After being on the waiting list for years, at long last I was invited to join the pack – the pack of high school and college instructors of mathematics – the few, the proud – who are AP Calculus Readers. I had heard it was a "really good professional develop experience." It was - the conversations, the connections, and finally a place to wear my very cool math T-Shirts without ridicule. However, it was not all sunshine and roses.

The Triangle Shirt Factory has nothing on this gig. Start grading at 8:00, coffee break mid-morning, lunch is an hour, afternoon break is 15 minutes again. Punch the clock – metaphorically of course – at five. In addition, you were never more than a minute's walk from caffeine – coming in yummy assorted forms: coke, coffee, brownies or little Mars bars. Keep 'em producing. More, more, more. Actually the official word – which was abided by – was it was all about accuracy, not speed. Whew! Nonetheless, as a first year grader I wanted to do good: do my share of work and be accurate. And I was all too happy to drink my share of coffee to make it happen.

I went to my first briefing – got the low-down on what was correct, what wasn't correct – and all the gray variations – the "valiant attempts" – which could mop up a point for all those potential – future – engineers' and physicists' high school AP calculus scores. The first briefing went smoothly. I got my first stack – asked my generous and experienced table partner Deb (the editor of this newsletter) lots of question to make sure I got it right. Then I found my cadence – clipping along – and my first positive feedback from the table leaders who were "backreading" (verifying) that I was grading things correctly. Yeah. I was good (enough). I thought I had it. Next day, next question. Briefing. Grade. Get "backread." Oh no. There were a couple of errors. I felt bad. We are all such children, I realized. We want to do well, get a pat on the back, and be at least just a little above average. We don't want to be reprimanded, told we are not doing something correctly. Hopefully no one noticed. How embarrassing. Phew – yes – on a couple of occasions it was I that was correct – the table leader missed it. An odd victory if you will. Nonetheless any error was not acceptable to me. I curtailed my cadence. I was determined to get it all correct. I slowed down and came up with my own system of "backchecking" my scoring – and the "backreaders" were happy again.

Reading AP exams was tiring and worthy of my clearing out my calendar for eight days. I would encourage all who are eligible to apply and take the opportunity when it comes around.

My First Year as a Table Leader
Todd Moyer - Towson University, Towson, MD

The reading as a first-year table leader was definitely a different experience. I am used to arriving one day before the reading, meeting up with friends, and getting in that first round of golf. Instead, I flew in three days before the reading and missed out on that round of golf.

The table leader group seems to be tight with each other, but they are extremely welcoming. My past table leaders were excited to see that I had now joined them. We spent our first two days being briefed on all six AB questions, going over a tentative rubric, gaining an understanding of the philosophy of the grading of each problem, and providing feedback on the rubric and grading methods. This is not finalized until the actual briefing with the table leaders.

To close the second day, my table leader partner and I set up the room. We decide the table partners and the seating arrangements around the table. We attempted to pair an acorn (a first year reader) with someone more experienced, a high school teacher with a college instructor, a male with female, and match pairs in terms of anticipated speed. We also considered the merits of some strategies, such as placing the faster readers in the front or rear of the room.

The biggest change in being a table leader was being considered an expert on scoring a problem. Instead of asking questions, I was now to answer such questions. In the beginning of the reading, I relied a lot on my table leader partner. She was incredible; I thought that we complimented each other very well. As time passed, I felt much more comfortable.

One of the unexpected perks of being a table leader is going for walks. We would walk to the question leader pod for answers to questions from readers that we were not able to provide. We also went to the QL pod for one book placed inside another book, offensive language, or other irregularities with books. These walks broke up the reading sessions and made time go faster.

Back reading is a different animal. I always thought that I was a somewhat speedy reader. During back reading, I was never able to find that "groove" that allows you to pick up that speed. I did not feel too uncomfortable taking my first folder back. I believe that my partner and I designed a room of teamwork, where we were all working together to read the exams efficiently and consistently. Usually, when I took some books back with questions, it was a chance for the reader to teach me the rubric. I would ask the reader to score the problem with me, thereby finding the differences in our scores. And trust me, it was not a given that I had the correct score. But it was in a spirit of cooperation where we both learned the rubric and philosophy better.

A table leader does have a different time schedule than the readers. We are not to leave the reading room until all readers have left, which shortens breaks. We also leave breakfast twenty minutes early to attend the daily table leader meeting with the chief reader. These meetings were very entertaining and informative. Here we learned the percentages of completion, the mean scores of the problems so far, and any announcements that we are to share with our reading rooms. I also got to see a less serious side of the chief reader.

One thing that I was touched by was the card signed at the end of the reading. As a reader, I never really was overly concerned about signing the card. I always would put in a caring thought, but figured that the card was just read and thrown away by the table leader. Boy, was I wrong! It felt good to read the compliments in the card. Through the card and conversation, the readers were pleased to be in our room. They felt like they were in a low stress environment. It was a relaxed setting, but a working room. This was evident when we took back minimal books with questions. There seemed to only be about two questions per stack of 25 books consistently. As a rule, my partner and I never felt like we had to back read more than one folder of any reader.

I was very pleased with my first year as a table leader. The readers in our room were fantastic, and I am not sure if I could have had a better partner to show me the ropes.

Being a Question Leader
Virge Cornelius - Lafayette High School, Oxford, MS

This was my second consecutive year to serve as a Question Leader (QL) for the AP Calculus exam (Common Problem AB5/BC5). I served as a reader, Table Leader (TL), and Question Team Member (QTM) prior to my work as a QL. A QL's main job is to train readers how to grade student work based on the standard (rubric). What readers see is a polished training session (or "briefing") with a miked QL, numerous slides to explicate the issues and how to award partial credit, and actual student solutions. What readers do not see is all of the "behind the scenes" work that goes into the briefing. About a week after the AP Calculus exam is administered, the Chief Reader (CR) lets the QLs know which of the nine problems will be theirs. Throughout the following weeks, more information comes to the QL. For example, the QLs learn the order in which we will grade the problems, who the team members are, and what some student solutions look like. By the time the QL boards the plane for the reading site, many questions are answered, but many remain unanswered and even unasked. AP Calculus QLs arrive on site for the "pre-reading" along with the CR and Exam Leaders (ELs) six days before the reading begins. The first day of meetings is spent in an intense round-table discussion where we spend about 45 minutes on each question, examining issues and giving suggestions. The second and third days of meetings the QL spends with the two team members, looking more closely at student work and selecting samples. During these Question Team meetings, the Team is visited by the CR and the ELs who really challenge the scoring rubric and ask a lot of "what ifs." The QL prepares slides and notes for the Table Leader briefing, which are on days 4 and 5 of the pre-reading.

What the QL has conceptualized as a road map for scoring gets tested and challenged at the Table Leader briefing. To be honest, this is the tougher than the "big" briefing. After the TL briefing, the QL is able, with the Team, to further hone and revise the presentation. About 36 hours before the "big" briefing, the final meeting takes place with just the QL, the ELs and the CR. The final scoring standard gets set and all of the issues, questions and comments that we can collectively think of are picked apart. Then the QL needs to refine the slides, rehearse for the "big" briefing, and participate in the reading by attending the opening session and other QLs' briefings and helping with folder and test irregularities in the QL work space, or "pod." After the QL briefs the readers who will be scoring his/her problem, the QL sits in the "pod" to answer questions from TLs whose rooms are working on the problem. At first there are a lot of questions as readers are getting on the standard, but then the number of questions decreases as the readers see more and more student work. About 1.752 days after the "big" briefing, the readers finish scoring problem and the QL finalizes reports for the CR and the College Board. The QL will also help in the QL pod with folder and test booklet irregularities. There will be folders not yet scored for the QL's problem (for whatever reason the folders were taken out of the flow, etc.), and the QL will score those as well. When all of the readers get dismissed on the final day of the reading, the QLs stay with the TLs, ELs and CR to grade all of the extra exams found on site. Then, it is time to say goodbye and head home!

Being an Exam Leader
Peter Atlas - Concord-Carlisle Regional High School, Concord, MA

It is the responsibility of the Exam Leader to assist the Chief Reader in setting the standards for the six free response questions. Standards consist of two columns: one in which we present what we consider to be a perfect response, the other a roadmap of how we're going to award partial credit for those students whose response falls short of perfection. The Exam Leader's main job is to ensure that the standards that we use are consistent among the different free response questions -- that is, that students earn the same number of points for skills regardless of the problem in which the skill appears. Exam Leaders assist Question Leaders in preparing their briefings, offering them guidance on how we've read similar problems in the past, and where the boundaries should be for the awarding of partial credit points. Exam leaders assist Question Leaders in the resolution of disagreements about the interpretation of the standard for their problems, and offer support and clarification, if necessary, during briefings to Table Leaders in advance of the main reading. If, when considering particular student work, a Reader cannot resolve whether or not to award a partial credit point, (s)he confers with his/her Table Partner. If, together, they cannot resolve whether or not to award the point, (s)he brings the student work to his/her Table Leader. If the issue is one that has not previously been covered in Table Leader briefings and the Table Leader cannot resolve whether or not to award the point, (s)he brings the student work to the Question Leader. In rare cases, the Question Leader is unable to resolve whether or not to give the point, and will ask the Exam Leader for guidance.

Exam Leaders give some logistical assistance during the reading, helping to insure that rooms are well-stocked with boxes of tests to read, that any personnel issues are dealt with professionally and in an appropriate manner, that ETS is informed whenever student work indicates that the student is a threat to him/herself or others, and when cheating is suspected. Finally, the Exam Leaders offer support to the Chief Reader, helping to ensure that the Reading proceeds smoothly, and that all student work is scored fairly and consistently by the end of the Reading session.

2011 Exam Question Discussions

The actual questions and rubrics can be found on AP Central. What follows is a discussion of how graders scored according to the rubrics and ideas that will help teachers in preparing students for the exams.

AB1 - Particle Motion
Carolyn Yacke - Mercer University, Macon, GA

In this problem, students were asked to consider the motion of a particle. The question dealt with the subtleties of velocity versus speed in part (a) and distances versus position in part (c). In part (b), students had to remember to find average velocity rather than just change in position. In part (d), distance traveled is insufficient because it does not account for the initial position of the particle at time $t = 0$. In other words, each part of the problem requires careful thought and attention to detail.

Part (a) — 2 points: Students were asked whether the speed of a particle was increasing or decreasing at time $t = 5.5$ and to give a reason for their answer. The desired answer is that for the function given, both velocity and acceleration are negative. Because they have the same sign the speed of the particle is increasing. Noting that the signs were negative, which could be done by reporting the correct values of $v(5.5)$ and $a(5.5)$ was important. The most common student mistake was to consider the sign of only one of $v(5.5)$ or $a(5.5)$, which is insufficient for determining whether speed is increasing or decreasing. Some students noted that $v(5.5)$ and $a(5.5)$ had the same sign, but failed to report that both were negative. This answer received only one of the two points.

Part (b) — 2 points: Students were asked to find average velocity. Many students understood the idea of writing the integral of v(t) from 0 to 6, which was worth 1 point. Many of those students went further and divided by 6 to earn the second point. Several common mistakes were various typos (copy errors and calculator entry errors), such as leaving off the +1 or inserting it into the argument of the sine function. Many fewer students than usual had their calculators in degree mode. Conceptually the most important and most common error made by students was that of reporting (v(6) - v(0))/(6 - 0) or the average rate of change of the velocity function rather than the average velocity of the particle. The distinction between these two concepts is confusing to students because of semantics and the relationship between them afforded through the antiderivative; therefore, particular attention to the similarities and differences is a suitable topic for class discussion. This issue also arises in AB4.

Part (c) — 2 points: Students needed to find the total distance traveled. One point was awarded for the integral and one point for the answer. The integrand needed to have absolute value signs around v(t) so that distance, rather than change in position, would be found. In order to take this approach, the students had to realize that they coud apply the absolute value function to an existing function in the calculator and subsequently integrate the result. An alternate approach frequently taken was to find the unique point at which the particle changes direction, referred to in part d, which is t = 5.19552, and calculate the integral from t = 0 to 5.195 of v(t) minus the integral of t = 5.195 to t = 6 of v(t). This second method allows the student unaware of absolute value capability of the calculator to proceed. In either case, the student should use the calculator to integrate on this calculator active problem, as the given function does not lend itself to integration by hand. Some discussion of the strategy of calculator use on the AP exam would have benefited the relatively small proportion of students who were convinced they had successfully found the antiderivative of the function and those who had difficulty hand calculating the zero of v(t). By far the most common error on this part of the problem was students forgetting the absolute values.

Part (d) — 3 points: Students were asked to find the position of the particle at which it changed directions. The first point was earned by setting v(t) = 0. The second was awarded for writing an appropriate integral representing the change in position of the particle between time t = 0 and the time of the direction change. The answer garnered the final point. Frequently students found the t-value for the turning point and the position change but forgot to account for the starting position of the particle.

AB/BC 2 - Tea and Biscuits
Dow Christenson – Desert Hills High School, St. George, UT

This problem was referred to as the tea and biscuit question (or the Royal Wedding). Given a tabular set of data, time (t) and temperature (H(t),degrees Celsius), students were asked to demonstrate multiple calculus concepts from the data and given equations.

Grading:

Part (a) - 1 point. Students are asked to "use the data" from the table to approximate the rate that the tea is changing temperature at time t = 3.5. To earn the point, students needed to "use the data." A difference quotient with data from the table, specifically (2,60) and (5,52), must have been clearly shown.

Part (b) - 3 points. Students were required to use trapezoidal sums to evaluate $\frac{1}{10}\int_0^{10} H(t)dt$. As is often the case for this type of problem, the intervals were not evenly distributed. This necessitated the use of a trapezoidal sum, not the trapezoid rule. The students then had to explain the meaning "in context" of the integral. In reading the explanation point, graders were instructed to look for three keys: average temperature, degrees Celsius (which could appear in the trapezoidal sum), and reference to the time interval. The second point was awarded when the students demonstrated that they were using trapezoids and summed them. They could break it into rectangles and triangles. A third appropriate method was to average the LRAM and RRAM. The third point was gained by giving the average over the interval, $52.95°C$.

Part (c) - 2 points. Students were supposed to demonstrate their understanding of the Fundamental Theorem. The first point was for giving the value of the integral, -23. This point was only given if that answer was in the presence of "43 - 66" or "H(10) - H(0)." The second point was stating correct meaning of the integral, that the temperature dropped by $23°C$ over the interval $0 \le t \le 10$. Just like part (b), students must have made reference to a drop in temperature, degrees Celsius, and the interval.

Part (d) - 3 points. First point awarded for the integrand, $B'(t)$. The second point was for evidence that the student used the initial conditions, $B(0) = 100$. The third point was given for the answer, 8.817.

Observations for teachers:

Part (a) - Students had trouble dealing with data in the table. A surprising number of them could not come up with the difference quotient. Some used wrong data values. Others switched the numerator and denominator. Many did not know exactly what was being asked and continued on to find the tangent line, approximating H(3.5) not H'(3.5).

Part (b) - Many students tried to use the Trapezoid rule instead of sums of trapezoids. When explaining, many students made reference to a rate of temperature, not the average.

Part (c) - There were many students who just gave the answer, -23, or $\int_0^{10} H'(t)dt = -23$. Neither of these got credit since they did not SHOW understanding of the FTC.

Part (d) - Students seemed to do better on this, maybe because they had an equation and felt more comfortable with this representation. One area in which they made errors in was using the initial conditions.

More of AB 2/BC 2
Vicki Carter - West Florence High School, Florence, SC

In this contextual problem, students were presented with a table containing data about the temperatures of a pot of tea as it cooled. The last part of the problem introduced biscuits that were removed from the oven and also cooling. This question appeared in the calculator-active part of the exam. Units were asked for in some parts of this problem. In part (a), students were instructed to approximate the rate at which the pot of tea cooled. In part (b), the students were to explain the meaning of the average value of the function, $H(t)$, over the 10 minutes and approximate the value with a trapezoidal sum. In part (c), the students were to evaluate a definite integral and explain the meaning of the expression. In part (d), the students had to find how much cooler the biscuits were than the pot of tea at a particular time.

Part (a) – 1 point. The point was awarded for setting up a difference quotient; therefore, the student had to show the work that led to the answer. The supporting work had to indicate that the student computed both a difference and a quotient using the data associated with t = 2 and t = 5 in the table of values. If students wrote $\frac{H(5)-H(2)}{5-2}$, they also had to write an equivalent numeric value using data from the table. $\frac{H(5)-H(2)}{5-2}$ without any other work would not earn the point. Units were ignored in this part of the problem even if the units were incorrect.

Part (b) – 3 points. The meaning of the expression with correct units was the first point. Students had to say that the expression represented the average temperature in degrees Celsius over the 10 minutes. The degrees Celsius could appear attached to the estimate. The 2nd and 3rd points were for the estimate using a trapezoidal sum. The trapezoidal sum point was a conceptual point. The reader had to be convinced that the student was working with subintervals of widths 2, 3, 4, and 1 along with the averages of the $H(t)$ values. Indication of finding a sum was also part of this point. The correct estimate was the third point. Students who attempted to use the Trapezoidal Rule did not earn the 2 trapezoidal sum points. Since this was a question about average temperature, many students neglected the $\frac{1}{10}$ as they set up their trapezoidal sum. Students often misrepresented equivalences with an equal sign whenever the $\frac{1}{10}$ reappeared in the problem.

Part (c) - 2 points. The first point was to evaluate $\int_0^{10} H'(t)\,dt$ using the Fundamental Theorem. The student had to present some work to indicate use of the Fundamental Theorem. This work could appear as $H(10)$ - $H(0)$ = –23 or 43 – 66. The students also had to explain the meaning of the expression in the context of the problem. The meaning had to include wording that indicated that the temperature changed in degrees Celsius from t = 0 to t = 10. Credit was generous for the meaning of the expression, since many students attempted to explain their answer as opposed to the expression. The degrees Celsius part of the explanation could appear attached to the estimate.

Part (d) - 3 points**.** The first point was earned with some indication that the student knew to integrate $B'(t)$. This was a conceptual point. In the absence of an integral sign, an expression for the antiderivative had to be clearly labeled as $B(t)$ and be correct. Use of the initial condition earned the student the 2nd point. The 3rd point was for answering the question. Ideally the student should write $B(10) = 100 + \int_0^{10} B'(t)\,dt$ and then evaluate this expression with the calculator. The difference in the temperature of the pot of tea and the estimated temperature of the biscuits could now be found. A number of students attempted to find the indefinite integral, label it as *B*(*t*), and use the initial condition to compute the constant of integration. There were some antiderivatives presented with no supporting work which resulted in the students earning 2/3 points on this part of the problem.

Summary - Students who were not well practiced in the use of tabular data had a difficult time with this problem. Teachers should work with these types of problems with students in both AB and BC Calculus. There are several problems of this type that can be found in past free response questions. Many calculations in this question required the communication of equivalent expressions as well as the correct answer. Misuse of equal signs occurred in both parts (b) and (d). Students need to make sure they communicate good mathematics in their presentation of answers. Overall, this was a good question for the students who were well-versed in tabular data problems.

AB 3 - The Puffin Problem

Karen Hastings, Freedom High School, Freedom, PA & Robert Morris University, Moon Township, PA

Part (a) - 2 points. To do the first part you need to show us that you are taking the derivative of *f*, getting $24x^2$, and plugging in 1/2 to get 6. Then you need to have a line that goes through (1/2, 1). I suggest you put it in point-slope form and stop! So long as you convince us that you took a derivative, even if your derivative is wrong, if your line goes through the point (1/2, 1) you will get 1 of the 2 points. So, $f'(x) = 24x^2$, $f'(1/2) = 6$, and $y-1 = 6(x-1/2)$.

Part (b) - 4 points. Make sure you have $\int [g(x) - f(x)]dx$ (preferred) or $\int [f(x) - g(x)]dx$. This second answer will give you a negative area and you will have to adjust it at the end. When you do this say "I got a negative area of $-\frac{1}{\pi}+\frac{1}{8}$, which is negative, so the answer must be $-\frac{1}{8}+\frac{1}{\pi}$." DO NOT PUT $-\frac{1}{\pi}+\frac{1}{8} = -\frac{1}{8}+\frac{1}{\pi}$!!!!! That is a false statement and you will have just thrown away a point. You get 1 point for either integral mentioned. Your second point is for having one antiderivative correct. The 3rd point is for having the rest of it correct: $-\frac{1}{\pi}\cos(\pi x) - 2x^4$. You have to have at least one of the two antiderivatives right for me to look for the answer and you must have a $\cos(\pi x)$ in the antiderivative. Your final answer must be positive.

So, the correct expression is $\int_0^{1/2} (\sin(\pi x) - 8x^3)dx = -\frac{1}{\pi}\cos(\pi x) - 2x^4 = -\frac{1}{\pi}+\frac{1}{8}$. Be careful when you are taking the antiderivatives, as the $\frac{1}{\pi}$ that goes out front should only be in front of the trig part, not the polynomial part. A lot of students put it in front of the whole expression and lost points. You might want to make it two separate integrals to be safe.

Part (c) - 3 points. The 1st point is easy - all you have to have is $\pi\int_0^{1/2} \textit{something}$. Make sure you have something after the integral sign and don't forget the π !!!!! The last 2 points are for both parts of $(1-f(x))^2 - (1-g(x))^2$. If you have them reversed, you lose 1 point. It is easier to use f(x) and g(x) instead of writing out the functions. If you put $(1-\sin(\pi x)^2$ you will lost a point because you are missing one) on the end !!! It stinks – I know - I am sorry! We were very picky about parentheses and equals this year.

So, the correct expression is $\pi\int_0^{1/2} \left[(1-8x^3)^2 - (1-\sin(\pi x))^2\right]dx$ Hint: On a non-calculator problem, if you find that you are doing a lot of arithmetic, you are probably on the wrong path! Again, remember 3 decimal places and don't simplify anything that you don't have to!!!!!!!

AB4/BC4 - The Graphiti Problem
Larry Peterson - Northridge High School, Layton, UT

This problem was another in a long line that tests a student's understanding of the Fundamental Theorem of Calculus. There was a unique twist in the problem. In addition to the typical problem defined as an integral of a displayed function, a linear term was added so that $g(x) = 2x + \int_0^x f(x)\,dx$. This caused many students problems because they couldn't connect the two ideas together properly. The committee used the graphical platform to see if students could use information from a graph to evaluate a function defined by an integral. In addition, three of the four sections required justification and explanation. This problem really showed which students knew their material.

Part (a) - 3 points. The first point came from evaluating the *g(-3)*. Most students handled the linear term but missed the directed area of the quarter circle. Since the integration was from right to left, the signed area needed to be negative. The next point came from finding $g'(x)$. Lots of students recognized that the Fundamental Theorem of Calculus should be applied to the integral, but they often forgot the linear term. This error came back to haunt them later on. After finding $g'(x)$, students were asked to evaluate $g'(-3)$. This meant they had to find *f(-3)* from the graph. Usually, they were successful if they got the second point.

Part (b) - 3 points. To find the absolute maximum value of *g*, students needed to set $g'(x) = 0$. If they missed the derivative in part (a), they were unlikely to find the critical value. After getting the critical value, most students missed the third point because they used a local maximum argument instead of a global maximum argument. It wasn't enough to show that the derivative changed sign; students needed to show there were no other points by appealing either to the domain along with increasing/decreasing behavior or by evaluating *g(x)* at the endpoints as well as the interior point. Few students were able to do this successfully.

Part (c) - 1 point. This was a simple point to earn if done correctly. Most students knew that $g''(x) = f'(x)$ and could reason the behavior for *g* based on the graph of *f* to recognize that the candidate for the inflection point occurred at x = 0. But the supporting explanation was often in error. The most common response was "There is a point of inflection at $x = 0$ because $g''(x)\,or\; f'(x) = 0$. First of all, $f'(0)$ was undefined because a corner appeared in the graph of *f(x)* at x = 0. Students had to communicate that the sign of $g''(x)$ changed at *x = 0*. This could be done by talking about $g''(x)$ or by discussing the increasing/decreasing behavior of the graph of *f*.

Part (d) - 2 points. The two points for this section were straightforward. The first point was given for computing the average rate of change through the endpoints of the graph of *f*. Many students did this correctly (there were some arithmetic errors). This year, students were required to list the actual y-coordinates $\frac{-3-(-1)}{3-(-4)}$ instead of referring to them by function notation: $\frac{f(3)-f(-4)}{3-(-4)}$. This was due to the apparent student confusion of the x- and y- coordinates of the right endpoint. Some students mixed them up. There were students who computed the correct value mentally and just wrote down the correct value $-\frac{2}{7}$. A bald answer did not earn any points. The instructions for the exam clearly stated that students must show their setup in order to earn the points for the answer. Sadly, many students misinterpreted the average rate of change to mean the average value and used a definite integral to obtain their incorrect result. This may have been due to the fact that an average value question was on an earlier problem. The second point came from recognizing that the conditions for the Mean Value Theorem weren't met. Students needed to declare that the function wasn't differentiable over the entire domain. Listing either the two points where the derivative failed to exist, $x = -3\,or\,x = 0$ directly, or just making a blanket declaration of non-differentiability at some point was sufficient.

Student referred to these points as cusps, corners, vertical tangents, etc. Any mention of an unusual situation was accepted. A blanket statement that said the function needed to be continuous and differentiable did not earn the point. Students had to specifically note there was a problem with the derivative.

There were several misconceptions among student answers. According to a large number of students, the MVT did not apply to piecewise functions. Others said the function was not continuous even though the stem clearly stated the function, *f*, was continuous. Some students thought the MVT meant there might be a point where the tangent line was parallel to the secant line, but such a point was not necessarily required.

AB5/BC5 - The Solid Waste Problem
Bob Angley - Charlotte Christian School, Charlotte, NC

This problem involved a landfill containing tons of solid waste. An increasing function W modeled the waste at the landfill.

Part (a) - 2 points. The student was asked to use the line tangent to the graph of W at t = 0 to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (t = 1/4). The first point was awarded for finding the value of dW/dt at t = 0. Many students experienced an immediate disconnect in this problem when the derivative was not in terms of the independent variable. They did not connect the 1400 tons to be the value of W when t = 0. Since they did not see a way to use t = 0, many of them tried to do part (c), solve for W as a function of t, in part (a). The second point was awarded for calculating the approximation at t = 1/4 using the line tangent to the graph at t = 0. While most calculus students usually can do a tangent line approximation, many did not see a way to evaluate the derivative at t = 1/4, and lost the second point. BC students made the correct correlation here more often than AB students did. A significant number of students substituted the zero for W and approximated the amount of solid waste to be 1397 (though they were given that W was an increasing function with an initial value of 1400).

Part (b) - 2 points. The student was asked to find the second derivative "in terms of W" and determine if the answer in part (a) was an underestimate or an overestimate of the amount of solid waste in the landfill at t = 1/4. Less than ten percent of the students did this successfully. Many students treated the W as if it were *t* and got the constant 1/25. Others who recognized the need to apply the chain rule frequently left the answer as 1/25 times dW/dt. These students lost the first point since the expression was not in terms of W. Students who did not get the first point were not eligible for the second point. To earn the second point in part (b), the student was required to indicate that the second derivative was positive and that the approximation was an underestimate of the amount of solid waste present at t = 1/4. Note that the second derivative is positive for the domain of the function [0,20], therefore the student was not required to include an interval for which the function was concave up. However, the student lost the second point if he used a local argument by declaring that the second derivative was positive at t equals some particular value. The requirement that the first point in part (b) must be earned in order to be eligible for the second point probably came from a desire not to reward a student who did not know to use the chain rule and got a second derivative that was positive for all values of *t*. The second point was not awarded for students who said, "The function is increasing at an increasing rate." One could argue that the student who had 1/25 times dW/dt should be eligible for the second point since they had applied the chain rule correctly and were told in the stem of the problem that W was an increasing function (so they could make an accurate deduction that the second derivative was positive). The reasoning for not making the student eligible was that they had not followed the directions completely (by getting the second derivative in terms of W).

Part (c) - 5 points. The student was given the initial condition W(0) = 1400 and asked to find the particular solution W = W(t) to the differential equation. The good news about part (c) was that students who did not understand parts (a) and (b) had a chance to score five points in part (c). The rubric awarded points in the same manner as past years for separable differential equations. The differential equation was not particularly hard to separate or antidifferentiate. Many of the students scored at least four of the five points. Among the BC students, most did well in part (c). The AB students were less successful. A plausible explanation would be that some AB classes may not have had time to practice initial value separable differential equations as often because of time constraints. The first point in part (c) was scored more leniently than in past readings. If a student made an error in the separating the variables but wound up with an expression that when integrated would produce a logarithmic expression in terms of W, the student lost the first point, but was eligible for points 2 - 4. This was a departure from the old rule that if the terms in the differentials were not separated correctly (or at least separated with a valid attempt) the student received 0/5. Students who substituted the initial values while the equation containing the antiderivatives still contained a log expression had more difficulty solving for W than those who exponentiated first and then substituted. Some students lost their eligibility for points 3 - 5 by inserting the constant too late.

Basic takeaways for teachers:

- This problem underscored the basic need that when teaching a new concept teachers need to use straightforward examples first to be sure the concept is understood on an elementary level, then proceed to more difficult examples (algebraically/conceptually), and also integrate the present concept to previously learned concepts.
- When separating differential equations students need to use multiplication and division rather than addition and subtraction.
- Part (b) stressed the need to teach students how to justify their answer "using calculus reasons."

I thought this problem raised the bar for writing equations for tangent lines. On past exams, many tangent line problems were very much alike: use the point of tangency and compute an easy derivative at the given domain value, then plug into the point-slope equation for a line and solve for y. This problem rewarded good thinkers and penalized all others. I also thought that students who used Leibniz's notation for derivatives regularly had a much better chance to think correctly in part (b).

AB 6 – Piecewise Problem
Janet Lewis, California Academy of Math and Science, Carson, CA

This problem involved a piecewise function for which students needed to prove continuity, find its derivative, the value x at which the derivative had a specific value, and find the average value of the function over a specific interval.

Part (a) – 2 points: In order to earn both points, the student had to use the definition of continuity to show that the limit exists, the function is defined at the point, and that the limit and the function value are equal. The most common error was that students assumed that showing the left-hand limit and right-hand limit were equal was proof that the limit existed and was sufficient to prove continuity. Many students did not present a concept of limit, for example

$$1 - 2\sin 0 = e^{-4(0)}$$
$$1 = 1$$

Another less common mistake was not including the argument in the limit. The following received no points, even though it was nearly a perfect response.

$$\lim_{x\to 0^-} = \lim_{x\to 0^+} = f(0) = 1$$

Had this student included $f(x)$ within the limit, full credit would have been earned.

Part (b) – 3 points: The student earned two points for a correct piecewise presentation of the derivative and one point for the correct value of x at which the derivative equaled -3. The function included a trigonometric piece and an exponential piece, for which students sometimes confuse derivatives and antiderivatives. Hence, sign and coefficient errors were common. Even though the problem specified that the derivative did not exist for x = 0, many students kept $x \le 0$ within the piecewise derivative. The = was ignored in the grading standard. Another error was not presenting the derivative as a piecewise function. Derivatives, correct or incorrect, were presented with or without "$f'(x) =$". Readers had to see two correct derivatives in order to award one point. For the final point, students needed to find the value of x for which $f'(x) = -3$. Most students who got this far were able to successfully calculate x (at least on the papers I graded). The most common error was not eliminating $\cos^{-1}(3/2)$ as an answer.

Part (c) – 4 points: The student was required to find the average value of f on the interval $[-1,1]$. One point was earned for splitting the integral into two parts. One point each was earned for the correct antiderivative. The final point was earned for the correct average value. The most common error was not recognizing that the piecewise function was split at x = 0, thus requiring the integral to be split into two parts

$$\int_{-1}^{0} (1 - 2\sin x)\ dx + \int_{0}^{1} e^{-4x}\ dx$$

Even if the integral was not split, we were able to read for the correct antiderivatives. As stated in part (b), some students confused derivatives and antiderivatives, so sign and coefficient errors were common. For the last point, we were able to read with the student as long as at least one antiderivative was correct and the limits were correct. Thus, the answer point could be earned for an "incorrect" value. The most common mistake was not dividing by 2. Another (heartbreaking) error was not distributing the negative sign to the term $\cos(-1)$.

BC 1 - Parametric Motion
Olga Cadilla-Sayres - Saint Mary's School, Raleigh, NC

This calculator active question was a fairly simple motion problem with a particle moving in the xy-plane. Students were given the parametric form of the x and y-components of the particle's position as differential equations with initial conditions. Overall, readers reported that the rubric was clear, consistent, and easy to apply. Students did generally well except for a few calculator issues.

Part (a) - 2 points: Students were asked to find the speed and the acceleration vector of the particle at time t = 3. Students earned one point for the correct value of the speed as long as they showed the set up. Most did well and used some form of $\sqrt{x'(3)^2 + y'(3)^2}$ to compute the right answer in the calculator. The other point was awarded for the acceleration vector in whatever form the student chose to express the answer as long as there was evidence of work, i.e. $(x''(3), y''(3))$. Points were not awarded when the student went directly to the calculator even if the answers were correct. A few students lost points for being in degree mode even though they were only penalized once. From that point forward, the answers were graded correctly even if computed in degree mode.

Part (b) - 1 point: Students were asked to find the slope of the line tangent to the path of the particle at time t = 3. Evidence of $\frac{dy}{dx}$ was necessary to earn this point and readers were encouraged to read backwards into part (a) to find convincing evidence of the ratio. Several students lost the point, even if they had the correct slope, for not showing an equation. A few continued in degree mode. Many students also found the equation of the tangent line even though the question only asked for the slope. Some readers felt that this was due to poor wording in the question. One common error occurred when students thought the speed and the slope were the same.

Part (c) - 4 points**:** Students were asked to find the position of the particle at time t = 3. Two points were awarded for each component, one for the integral and one for the correct answer. Most students had no trouble finding the correct answer for the x-component even if they did not use their calculator. The integral $\int_0^2 (4t+1)\,dt$ can be easily done by hand and, since x = 0 at t = 0, they successfully arrived at the correct answer even if there was no evidence they were considering the initial condition. The first point was lost when no set up was offered, and a few did not earn the second point when they wrote their final answer as a conditional equation, i.e. $2t^2 + t + C = 21$. However, the integral involved in finding the y-component of the position, $\int_0^2 \left(\sin\left(t^2\right)\right) dt$, cannot easily be done by hand,. Nevertheless, many students tried. Readers reported seeing a variety of very creative ways, mainly integration by parts, of finding the antiderivative, but with no success. Some students remembered to use the calculator to solve the definite integral, which earned them a point, but did not include the initial condition, y = -4 at t = 0. A few continued working in degree mode.

Part (d) - 2 points: Students were asked to find the total distance traveled over the interval $0 \le t \le 3$. One point was given for the definite integral and the other for the answer. Bald answers, if just 21.091, received no points. Parentheses were an issue for some students. If the answer was incorrect but the integral had the correct parentheses, the student earned the first point; however, if the answer was incorrect but the parentheses were placed incorrectly, the student lost both points. A common error occurred when students found the arc length instead of the total distance traveled. A few continued their work in degrees.

General comments:

- Teachers should allow and encourage students to use calculators to compute definite integrals that cannot be solved by hand or cannot easily be solved manually. Students should be aware of the appropriate use of a calculator on the calculator portion of the free response section.
- Students need to communicate what they are doing or they risk losing a significant number of points.
- Many students with a clear understanding of the question lost points for easily avoidable errors like not expressing answers to three decimals places, working in the wrong mode, and/or not showing the set ups.

BC 3 - Area/Volume
Christine Kuzdzal - Guilderland Central High School, Guilderland, NY

In this problem students were given the graph of a region R, bounded by the x- and y-axes, the line x = k and the function $f(x) = e^{2x}$. In part (a) students had to set up an integral expression for the perimeter of R in terms of *k* which required the application of arc length. Part (b) was a classic volume of rotation about the x-axis in which students had to find the volume in terms of *k*. Part (c) was a related rate problem in which V changed as *k* changed. Students had to apply the chain rule to determine $\frac{dV}{dt}$. This was a non-calculator problem.

Part (a) – 3 points: The first point was for $f'(x)$, the second point was for the integral, and the third point was for the answer. The first point could be earned by either explicitly declaring $f'(x) = 2e^{2x}$ or by using it in the arc length formula. The second point was earned by having a correct definite integral with a correct $f'(x)$ or an incorrect but declared $f'(x)$. The third point was earned by adding $1 + k + e^{2k}$ to $\int_a^b$ any expression with a = 0 or 1 and b = k or e^{2k}.

Some students used area instead of arc length in this problem. Perhaps this was because they are so used to the area-volume combination problems. Students who applied arc length to this problem had the following common errors:

- Incorrect limits of integration (1 to k)
- Forgetting to square $f'(x)$ or incorrectly squaring $f'(x)$
- Incorrect $f'(x) = e^{2x}$, instead of $f'(x) = 2e^{2x}$

Part (b) – 4 points: The first point was for the integrand, the second point was for the limits, the third point was for the antiderivative, and the fourth point was for the answer. Once earned the first point could not be lost. Students earned this point even if they incorrectly declared that $\left(e^{2x}\right)^2 = e^{4x^2}$ and used it as the integrand. This simplification error was taken off the answer point. The limits point was earned only if both limits were correct. Students could not import incorrect limits from part (a). Once earned this point could not be lost. Students were eligible for the third point only if their integrand was correct or their integrand was of the form $e^{ax+b} + c,\ a \neq 0$ and c is treated as a constant when antidifferentiating. Students were eligible for the answer point only if they had earned the limit point and their antiderivative was one of the following: $\frac{1}{4}e^{4x},\ 4e^{4x}$, or e^{4x}. A missing π or an incorrect simplification came off of the answer point. When students tried to antidifferentiate and evaluate in one step, they needed to be correct or they did not earn the last two points.

Shells - The use of shells was permitted but required the sum of two volumes. If the students had just one volume they earned 0/4 points. To earn the integrand point the students had to have both integrals correct (πk could replace the equivalent integral). To earn the limit point the students had to have both sets of correct limits (πk could replace the equivalent integral). To earn the antiderivative point, the students had to have $\pi\left[k + ky^2 - \frac{y^2}{2}\ln y + \frac{y^2}{4}\right]$ or an equivalent expression. To be eligible for the answer point, they had to have earned the antiderivative point.
Some students had difficulty antidifferentiating $\left(e^{2x}\right)^2$ and applied the power rule to this exponential function. Other common errors were:

- Mishandling the coefficient
- Simplifying the integral incorrectly to e^{4x^2} and trying to integrate it
- Mishandling the distribution of π/4
- Evaluating e^0 as 0
- Not evaluating the integral, perhaps confusing this with the directions in part (a)
- Placing their work for part (b) in part (c) and receiving no credit for it

Part (c) – 2 points: The first point was for applying the chain rule and the second point was for the answer. The first point was a conceptual point and readers were looking for evidence of the correct use of $\frac{dk}{dt}$ everywhere it was needed. Mistakes in calculating $\frac{dV}{dt}$ did not come off of this point, but instead were taken off of the answer point.

Students could earn this point implicitly through their work to find $\frac{dV}{dt}$ or by stating $\frac{dV}{dt} = \frac{dV}{dk}\cdot\frac{dk}{dt}$ or $\frac{dV}{dk} = \frac{\frac{dV}{dt}}{\frac{dk}{dt}}$.

Students needed to use their V from part (b). Readers read for consistency with a student's incorrect V from part (b). To be eligible for the second point, st1udents must have earned the first point. To earn the second point their answer must be show appropriate use of 1/2and 1/3 and be correct or consistent with their correct derivative of an incorrect V from part (b).

Note to Teachers: Many students correctly and elegantly used the FTC to get from part (b) to part (c). Common errors were:

- Not applying the chain rule
- Differentiating some V other than their V from part (b)
- Incorrectly claiming the derivative of a constant is something other than 0.

Overall, students did well on this problem.

BC 6 - Infinite Series Problem
Gary Thompson - Grove City College, Grove City, PA

Scoring

Part (a) - 3 points. The first point was for the correct first four terms of the Taylor series for sin x about x = 0. The remaining two points were for the series for sin (x^2).

Part (b) - 3 points. The first point was for the correct first four terms of the Taylor series for cos x about x = 0. The remaining two points were for the using in part (a) and the cos x series to find the first four nonzero terms of the Taylor series for the given function *f* about x = 0.

Part (c) - 1 point. The students had to find the correct coefficient of x^6 in the Taylor series for the given function *f* which if done correctly from correct work in part b is -121. Some students did this the long way with 6 differentiations.

Part (d) - 2 points. The 1st point was for the form of the error bound while the 2nd point was for correct analysis.

Observations for Teachers:
I was on the question team for BC 6 and had an opportunity to view an exceptional number of books looking for particular issues and details. Here are some random observations:

I. With regard to content, it should have been fairly easy to earn the first five points in the problem: a student would only need to know the series for sin x, cos *x*, how to simply substitute x^2 in for x in the series for the sine function, and indicate at all that they were adding the series for sin x^2 to the series for cos x. I was hoping that 80 to 90 percent of the books would earn those first five points. Unfortunately, that was not the case.

The BC curriculum has a large number of topics, and I am afraid that infinite series often gets short-changed. I routinely spend 5 or 6 weeks on the topic in my class and hope teachers can find a way to spend a commensurate amount of time on the topic in their classroom as well. Some helpful hints: The sine function is an odd function, and contains only odd exponents in its series expansion. Moreover, cos x is the derivative of sin x, which provides an easy way to remember two series for the price of one. If you ignore the alternators and add the series, you get the series for e^x. This is equivalent to the statement that e^{x}= cos *Ix* + (*1/i*)sin *ix*. This quickly leads to Euler's equation, e^{ix} = cos *x* + *i* sin *x*and the corresponding exponential versions of sine and cosine: cos x = $(e^{ix}+e^{-ix})/2$ *and* sinx = $(e^{ix}-e^{-ix})/(2i)$. Note that the last pair of equations now imply that cos x = cosh*ix* and sin x = *i* sinh*ix*.

The last point in part (b) was earned by combining terms to get a true Taylor polynomial for f(x): this was a subtle point since we frequently encourage students to **not** simplify expressions, but recognizing that the coefficient of x^n is $f^{(n)}(a)/n!$ in a proper Taylor polynomial is necessary to earn both the last point in part (b) and the single point available in part (c). The two points in part (d) were for knowing a formula for the error term when using a Taylor polynomial to approximate a function at a given point. I can sympathize with students and teachers alike for having trouble with this topic, for it is typically the last topic covered in a grueling course. Kudos to the teachers and students who managed to earn one or two of the points in part (d)!

II. Mechanical issues: There were several spots in the problem where students failed to earn available points due to mechanical issues. In part (a), students generated some strange expressions when using the series for sin x to present the series for sin x^2. Notable errors included squaring the exponents rather than doubling the exponents, doing strange things with the denominators rather than just leaving them alone, and changing several minus signs to plus signs. I also noticed a handful of puzzling expressions in part (b) that were supposed to represent sin x^2 + cos x. A number of students multiplied terms together pairwise rather than adding. It is easy to make careless errors in the heat of the moment, but it was sad to see students losing points that they had in their pocket.

III. Test-taking issues: I saw, on a number of books, work in part (a) that earned no points at all and then a perfectly correct and labeled expression for sin x^2 in part (b). Unfortunately, the grading rubric would not allow the readers to award any of the three points in part (a) in those books. The grading rubrics in general have become more flexible with respect to awarding work in the wrong place, but the best strategy is to provide the answers in the space allocated for that part of the problem. Just as bad, I saw a number of books where the student provided the series for sin x^2, but not for sin x, in part (a). Please emphasize to your students that each question/statement in the test that requires an answer should be answered explicitly, in the appropriate location in the test booklet.

Items of Interest

The **NCCTM conference** will be held at the Koury Convention Center in Greensboro on October 27th and 28th. NCA^{2}PMT will hold a meeting on Thursday; you will need to check for exact room location and time. Discussions concerning a few of the questions on the 2011 AP exam will be held. The conference is an opportunity for learning new ideas for enriching your classes.

NCAAMPT Calculus Challenge http://courses.ncssu.edu/math/POW/POWindex.htm *Dan Teague*

New TI-Nspire CX (Color)

Graph differential equations: Model real-world events like population growth. Visualize and explore natural phenomena. You can graph and explore slope fields, direction fields.
Energize Statistics class: Study data like a baseball pitcher's earned-run-average over several years. More ways to view and analyze data in bar charts and histograms. Create summary-level frequency plots, probability distributions and clustered bar charts.
Graph in 3D: Graph and explore functions in a 3D space with the powerful 3D graphing functionality, ideal for science subjects and higher mathematics such as Pre-Calculus and Calculus.
Professional Development Opportunities : T³ (Teachers Teaching with Technology) International Conference is going to be in Chicago, IL March 2-4, 2012. Plan on being there to see the latest technologies and lessons – and to hear from your colleagues about what has been working in their classrooms.

TI-Nspire training opportunities include:

Getting Started – High School Mathematics: August 15-17 in Goleta, CA, Worcester, MA, Portage, MI;
August 23-25 in Surrey, BC; August 24-26 in Calgary, BC

TI-Nspire for Intermediate Users – High School Mathematics: August 15-17 in Worcester, MA;
August 22-24 in Rochester, NY

Getting Started Connecting Science and Mathematics: August 22-24 in Rochester, NY

For more information and to register go to http://education.ti.com/calculators/pd/US/

ONLINE AP Calculus Competition: Approximately 3500 students from 27 states and 5 countries participated in October and November. For more information, teachers can contact me at pross@gcbe.org The competition is free! Next year the competition starts the first week of November - registration process in late September. *Paul Ross*

Websites

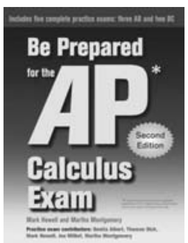
Course related websites: http://apcentral.collegeboard.com/calculusab
http://apcentral.collegeboard.com/calculusbc
To search the list archives for previous posts go to http://lyris.collegeboard.com/read/?forum=ap-calculus

Second Edition of Rogawski Text

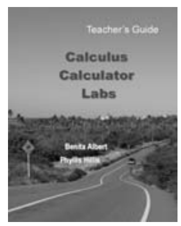
Calculus for AP® (2nd Edition) by Jon Rogawski and Ray Cannon

The new Rogawski book was highlighted at a reception sponsored by BFW publishers for AP Readers at the Westin Hotel in Kansas City. This new book is now co-authored by former Chief Reader, Ray Cannon (Baylor University). The exciting part is that not only have improvements been made in the text, but **both multiple choice and essay AP type questions have been included at the end of each chapter** (except the initial one on Pre-Calculus topics). There is an Early Transcendental version. Lin McMullin wrote the Teacher's Resource Binder. There is a book companion site at www.bfwpub.com/highschool/rogawskiforAP Rogawski, his wife, Julie, and Cannon were in attendance to discuss the text and sign autographs. For more info - www.whfreeman.com/rogawski2e

New Books Published by Skylit Publishing



Mark Howell and Martha Montgomery book
Second Edition



Benita Albert and Phyllis Hillis Calculator
Lab book - This is the teacher's edition.
There is a student lab book.

800 Questions in Calculus with Solutions is now e-edition only. The descriptions of the books are at www.skylit.com/beprepared-calc.html and http://www.skylit.com/calclabs.html. The order form is at www.skylit.com/orderform.pdf.

MAA Publication

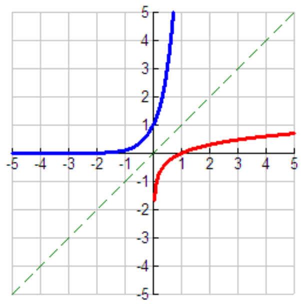
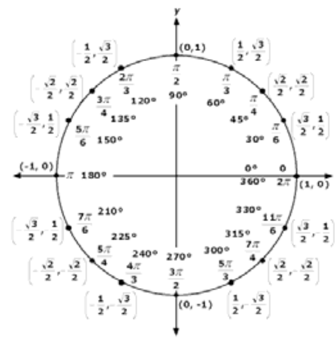
The Calculus Collection: A Resource for AP and Beyond*
Caren Diefenderfer and Roger Nelson, Editors
AP teachers and 2-Year college teachers get the special member price of $59.95. Call 1-800-331-1622. Use the promo code CCE/AP2. This is a 527 page hardbound book published in 2009. It consists of 123 articles selected by six veteran high school teachers, each of which was originally published in other magazines and journals. The articles focus on engaging students and provide alternate explanations of difficult ideas, providing opportunities for students to dig deeper into mathematical concepts.

Corrections to Winter 2011 Newsletter

Karen Hastings was perusing our website and looking at old newsletters when she noted that the answer to part 2 of the oil leak problem (she agreed with the other 11 parts of this great problem) was incorrect. The correct solution should have shown 3*1.3 + 3*1.9 + 18*2.7 + 1*12.8 + 3*13.5 = 111.5. When I contacted problem authors, Rhea Caldwell and Trish Morris, they commented that the problem had undergone several iterations of change and that in our newsletter version, the values in the table given had not changed to correspond to the changes they made to the problem. We apologize for any inconvenience this might have caused.

A PreCalculus Resource

The Stepping Stones to Calculus
A Comprehensive Guide to the Mathematics You Need to Know

TEACHERS: This resource reviews all the major pre-calculus topics beginning with integer operations. This book covers all the material your students need to know before they start their study of Calculus.
The Stepping Stones to Calculus: A Comprehensive Guide to the Mathematics You Need to Know *by* ***Sharon Cade ©2011*** *website:* ***steppingstonestomath.com*** *e-mail:* ***steppingstonestocalculus@gmail.com*** *$24 includes postage and handling – plus any applicable tax. $20 each if you order 10 or more copies (includes postage and handling, plus applicable tax.) See website for payment/paypal information. Color throughout the 156 page book to enhance understanding.*

UPDATES - Calculus In Motion & Algebra In Motion

Animations to the 2011 Free Response Questions from the AB & BC AP Calculus Exams (main versions) have been added to the *Calculus In Motion*^{TM} collection. If you already have *Calculus In Motion*^{TM}, simply send an email *along with your serial number from the front of your CD* and your name, etc., to Audrey Weeks at amweeks@aol.com. If possible, use a private email address as most school servers will block the attachment sent back to you. If you are within 2 years of your purchase date, these updates are free to you. If it has been longer, the fee is $30 (a bit more if you have an expanded license). This fee covers the next 2 years of new releases and modifications. The update order form can be downloaded from www.calculusinmotion.com. If you are not already a user of Calculus In Motion, wait no longer! This collection of 182 interactive animations spans the entire year of calculus – graphing, limits, derivatives, integrals, theorems, related rates, optimization, areas, volumes, Newton's Method, slope fields, Euler's Method, Maclaurin/Taylor series polynomials, and more! Simply visit www.calculusinmotion.com, link to "Order Form" and email, fax, or send your order ASAP so you'll be able to begin using the files with your students from week 1 of the upcoming school year. For *Algebra In Motion*^{TM} users, look for another update late this summer or early fall. When it is released, an announcement will appear on the www.calculusinmotion.com website with more details.

2002 - 2011 AB EXAMINATION
FREE-RESPONSE QUESTION FREQUENCY

Trish Morris, Greensboro Day School, Greensboro, NC

FUNCTIONS

	‘11	‘10	‘09	‘08	’07	‘06	‘05	‘04	‘03	’02
Zeros										
Asymptotes				X						
Symmetry										
Domain						X				
Odd/Even										
Range										
Inverse										
Limits	6			X						
Linear Equation	3,5									
Continuity	6				X				X	X

DIFFERENTIAL CALCULUS

	’11	’10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	’02
Analyze a Function Given as a Table of Values	2	X	X	X		X	X		X	
Tangent Line Equation	3,5	X	X	X	X	X	X		X	X
Differentiation & Evaluation	5,6	X	X	X	X				X	X
Increasing & Decreasing Functions		X		X	X			X	X	X
Critical Numbers, Maximum & Minimum Points – Relative & Absolute	4	X	X	X	X		X	X		X
Concavity				X	X		X		X	X
Inflection Points	4	X		X	X		X			
Average Rate of Change	4	X	X			X	X	X		

DIFFERENTIAL CALCULUS

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	’02
Critical Numbers, Maximum & Minimum Points, Increasing & Decreasing, Concavity, Inflection Points from the graph of $f'(x)$	4	X	X	X		X		X	X	X
Critical Numbers, Maximum & Minimum Points, Increasing & Decreasing, Concavity, Inflection Points from a table of values of $f(x), f'(x), \& f''(x)$				X			X			X
Curve sketching or analyzing data using information from a table of values of $f(x), f'(x), \& f''(x)$ **or from the graph of** $f'(x)$	4	X	X	X		X	X			X
Mean Value Theorem for Derivatives	4			X	X		X			X
Implicit Differentiation								X		
Linear Approximation	5	X	X		X		X			X
Related Rates		X		X	X				X	X

INTEGRAL CALCULUS

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	‘02
Area and/or Interpretation	2,3	X	X	X	X	X	X	X	X	X
Riemann Sums using Left, Right, Midpoint Evaluation Points			X		X	X			X	
Properties of Integrals		X	X	X					X	X
Trapezoidal Rule/Approximation	2	X		X			X			
Fundamental Theorem of Calculus	1-3	X	X	X	X		X		X	X
Mean (Average) Value	1,2,6	X	X			X	X	X	X	
Volumes of Solids: Disks and Washers	3	X			X	X	X	X	X	X
Volumes with known Cross Sections		X	X	X	X			X	X	

INTEGRAL CALCULUS

	‘11	**‘10**	**‘09**	**‘08**	**‘07**	**‘06**	**‘05**	**‘04**	**‘03**	**‘02**
Solving Differential Equations: Separation of Variables	**5**	**X**		**X**		**X**	**X**		**X**	
Drawing Slope Field from Differential Equation				**X**		**X**	**X**	**X**		
Rectilinear Motion: Equation(s) for Position, Velocity, & Acceleration; Direction of Motion; Total Distance	**1**		**X**		**X**	**X**	**X**	**X**	**X**	**X**
Rectilinear Motion: Position, Velocity, & Acceleration; Total Distance from the Graph of Velocity			**X**	**X**			**X**			
Rectilinear Motion Analysis from a Graph			**X**	**X**						
Definite Integral as an Accumulator	**1,2**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
Accumulation of the Derivative with Initial Condition	**2**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	

CALCULATOR

	‘11	**‘10**	**‘09**	**‘08**	**‘07**	**‘06**	**‘05**	**‘04**	**‘03**	**’02**
Draw a Graph in a Given Window					**X**		**X**			**X**
Find the Zeros of a Function	**1**									**X**
Find the Intersection Points of Two Graphs				**X**		**X**	**X**	**X**	**X**	
Evaluate a Definite Integral	**1,2**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
Evaluate a Derivative	**1**		**X**				**X**	**X**	**X**	**X**

2002 - 2011 BC EXAMINATION
FREE-RESPONSE QUESTION FREQUENCY

Trish Morris, Greensboro Day School, Greensboro, NC

FUNCTIONS

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	’02
Domain & Range				X						
Odd/Even										
Intercepts										

LIMITS & CONTINUITY

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	‘02
Finite Limits					X					
Limits at Infinity Infinite Limits						X				
Definition of Continuity										

DIFFERENTIAL CALCULUS

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	’02
Analyze a Function Given as a Table of Values	2		X	X			X			
Tangent Line Equation	5		X	X	X		X		X	
Differentiation & Evaluation	3,5			X						
Increasing & Decreasing Functions				X	X				X	X
Critical Numbers			X	X	X	X				X
Concavity				X	X		X		X	X
Inflection Points	4		X	X					X	
Average Rate of Change	4		X			X	X	X		X
Extreme Values	4		X	X	X	X	X			X

DIFFERENTIAL CALCULUS

	‘11	‘10	‘09	‘08	‘07	‘06	‘05	‘04	‘03	’02
Higher Order Derivatives					X	X				

Optimization Problems: Maximum & Minimum; Relative and Absolute						**X**	**X**			**X**
Curve Sketching or analyzing data using information from a table of values of $f(x), f'(x), \& f''(x)$ **or from the graph of** $f'(x)$	**4**		**X**	**X**			**X**			**X**
Mean Value Theorem for Derivatives	**4**			**X**			**X**			
Implicit Differentiation						**X**	**X**	**X**		
Linear Approximation	**5**		**X**	**X**	**X**					
Related Rates	**3**				**X**				**X**	

INTEGRAL CALCULUS

	'11	**'10**	**'09**	**'08**	**'07**	**'06**	**'05**	**'04**	**'03**	**'02**
Area and/or Interpretation	**2**		**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
Riemann Sums using Left, Right, & Midpoint Evaluation Points			**X**		**X**	**X**				
Properties of Integrals			**X**	**X**					**X**	**X**
Trapezoidal Rule/Approximation	**2**			**X**			**X**			
Fundamental Theorem of Calculus	**1,2,3**		**X**	**X**			**X**		**X**	**X**
Mean (Average) Value	**2**		**X**			**X**	**X**	**X**		**X**
Volumes of Solids: Disks and Washers	**3**				**X**	**X**	**X**	**X**	**X**	**X**
Volumes with known Cross Sections				**X**	**X**				**X**	
Rectilinear Motion: Equations for Position, Velocity, & Acceleration; Direction of Motion; Total Distance							**X**			

INTEGRAL CALCULUS

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Rectilinear Motion: Position, Velocity, & Acceleration; Total Distance from the Graph of Velocity			X	X			X			
Rectilinear Motion Analysis from a Graph			X	X			X			
Definite Integral as an Accumulator	2		X	X	X	X	X	X	X	X
Arc Length, Surface Area, and Work	3									

CALCULATOR

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Draw a Graph in a Given Window					X					
Find the Intersection Points of Two Graphs				X		X	X		X	
Evaluate a Definite Integral	1,2		X	X	X	X	X		X	X
Evaluate a Derivative	1		X					X		X

METHODS of INTEGRATION

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Integration by Parts				X	X					
Integration by Partial Fractions										
Improper Integrals						X				

DIFFERENTIAL EQUATIONS

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Solving Differential Equations by Separation of Variables	5		X	X		X			X	
Logistic Differential Equations				X				X		
Slope Fields				X			X			X
Euler's Method			X	X		X	X			X

SEQUENCES and SERIES

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Tests for Convergence							X			
Geometric Series										
Alternating Series & Error Approximation					X				X	
p-Series										
Manipulation of a Power Series	6		X			X		X	X	X
Power Series: Radius of Convergence										
Power Series: Interval of Convergence			X			X	X			X
Maclaurin Series	6			X	X				X	X
Taylor Series			X	X	X	X	X	X		X
Lagrange Error Bound	6			X				X		

PARAMETRIC, POLAR, and VECTOR FUNCTIONS

	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02
Parametrically Defined Curves								X	X	X
Derivatives of Vector & Parametrically Defined Curves	1		X			X		X	X	X
Tangent Lines to Parametrically Defined Curves	1							X	X	
Arc Length Including Parametrically Defined Curves	1		X							
Velocity, & Acceleration Vectors for Motion on a Curve	1		X			X		X		X
Polar Coordinate Graphs					X		X		X	
Area of Polar Curves					X		X		X	

Minutes
North Carolina Association of Advanced Placement Mathematics Teachers
Meeting of the Board of Directors
Panera Bread, Lawndale Drive, Greensboro, NC
July 9, 2011

Members Present: Martha Ray, Jeff Lucia, Gloria Dupree, Solomon Willis, Susan Jones, Roberta Rohan, Jim Pielow, Dan Teague, Deborah Britt
Members Absent: Steve Davis, Emogene Kernodle

1. Martha Ray called the meeting to order at 10:10 a.m. Introductions of board members attending their first meeting were made (Jim Pielow, Roberta Rohan, Solomon Willis, Susan Jones).
2. Board Meeting minutes of the June 26, 2010 meeting were distributed. Approval for the minutes had been obtained by internet following the 2010 meeting, The approved minutes were included in the Summer 2010 newsletter.
3. The Membership/Treasurer's Report was presented by Jeff Lucia. Both of these reports follow at the end of these minutes. It was noted that our membership has seen small drops over the past few years but we remain financially stable. The grant for 25% of our newsletter costs provided by University of North Carolina at Charlotte STEM Center (David Pugalee, Director) is a major reason for our fiscal solvency.
4. The 2012 AP Reading will be in Kansas City - both AP Calculus and AP Statistics will be located there during the same time. The 2011 Reading was Mike Boardman's last reading. Steve Kokoska will be the new AP Calculus **C**hief Reader. The 2012 AP Calculus exam will continue to have no penalty on the multiple-choice and the free response will continue to have a 2 GC, 4 non-GC split. An evaluation of these changes will follow.
5. The Summer 2011 Newsletter is almost complete. All articles are in now with final edits to follow prior to mailing.
6. The NCCTM Conference request was made and acknowledged on January 31, 2011 for the upcoming State Mathematics Conference in Greensboro at the Koury Convention Center. The request was made to present on Thursday, October 27^{th}, immediately following or preceding the Statistics session. Final confirmation of date, time, and room number have not been received.
7. A review was made of the terms of all Officers and board members. Below are the names, offices and expiration/renewal dates for each member:

    Deb Britt – Executive Secretary, permanent member
    Jeff Lucia – Treasurer/Membership Chair, permanent member
    Stephen Davis – Web Master, permanent non-voting member
    Gloria Dupree – Past President – Western Region, 2012
    Martha Ray – President – Central Region, 2014
    Emogene Kernodle – President Elect – Central Region, 2016
    Dan Teague – (expires 2012) – Eastern Region
    Jim Pielow – (expires/renewal 2013) – Central Region (may serve through 2017)
    Roberta Rohan – (expires/renewal 2013) - Central Region (may serve through 2017)
    Solomon Willis – (expires/renewal 2013) – Western Region (may serve through 2017)
    Susan Jones – (expires/renewal 2013) – Western Region (may serve through 2017)
    Vacant – (expires/renewal ______) – Eastern Region

    Requests were made for possible Eastern Region representatives and they will be contacted.
8. Dan Teague volunteered to continue the NCAAPMT online mathematics competition for another year. Most schools involved are not NC schools so an evaluation of this opportunity will occur at the next board meeting.
9. The 2012 Board Meeting was tentatively set for Saturday, June 23, 2012 in Greensboro.
10. President Martha Ray adjourned the meeting shortly before noon. Most board members stayed to enjoy a Dutch Treat lunch at Panera Bread.

Respectfully submitted by Deborah Britt, Executive Secretary

2011 Treasurer's Report

Submitted by Jeff Lucia, Treasurer/Membership Chair, July 9, 2011

Balance as of 6/26/10	5468.29
Deposits (memberships)	1425.00
Deposits (newsletter grants)	296.19
TOTAL	$7189.48
Newsletter (August 2010)	622.71
Newsletter (February 2011)	562.03
Balance as of 7/9/11	6004.74
TOTAL	$7189.48
TOTAL INCOME	1721.19
TOTAL EXPENSE	1184.74
NET GAIN 2010-2011	$536.45

The above gain for 2010-2011 represents our eighth consecutive year with an operating surplus. The following reasons can be cited:

1. We continue to receive a grant for each newsletter and mailing through Dr. David Pugalee from the UNC Charlotte Center for Mathematics, Science and Technology Education (CMSTE). These grants defray approximately one-fourth of our expenses. We express our sincere thanks to Dr. Pugalee and the CMSTE for their continued generous support.
2. We continue to hold down the cost of printing and mailing our semi-annual newsletter.
3. Numerous members pay their dues for some years in advance of the current year.

2010 Membership Report

Submitted by Jeff Lucia, Treasurer/Membership Chair, July 9, 2011

Current membership is 283, including 71 whose membership expired 3/2011 who will receive a reminder but will not get the summer newsletter unless they renew. This overall number is down 38 from 321 last year at this time, and the number of paid up members is currently 212, as opposed to 225 last year. Membership is down in most areas: Eastern (-2), Central (-2), Western (-4), other states (-30). This is a continuation of a somewhat disturbing trend of several years, but we are still financially viable. We continue to send courtesy copies of the newsletter to non-member authors of contributed articles. A breakdown of our membership is as follows:

North Carolina		Other States			
Eastern -	23	California	23	Alabama	2
Central -	42	New York	19	Arizona	2
Western -	40	Virginia	14	Kentucky	2
Total	105	New Jersey	12	Nevada	2
		Georgia	10	Wisconsin	2
		Colorado	9	Hawaii	1
		Texas	9	Idaho	1
		Florida	8	Illinois	1
		South Carolina	8	Indiana	1
		Washington	7	Kansas	1
Foreign		Massachusetts	6	Mississippi	1
		Pennsylvania	6	Missouri	1
Brazil -	1	Tennessee	5	New Mexico	1
Canada -	1	Maryland	4	Oklahoma	1
Israel -	1	Ohio	4	Utah	1
Turkey -	1	Connecticut	3		
Total	4	Michigan	3		
		Minnesota	3	District of Columbia	1

TOTAL OTHER STATES - 174 (NC plus 33 states & DC)

None from 16 states: Alaska, Arkansas, Delaware, Iowa, Louisiana, Maine, Montana, Nebraska, New Hampshire, North Dakota, Oregon, Rhode Island, South Dakota, Vermont, West Virginia, Wyoming