

The North Carolina Association Of Advanced Placement* Mathematics Teachers Newsletter

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Vacant position

Web Address

www.ncaapmt.org/calculus

Notes from the President's Desk

I recently had knee replacement surgery and am now recovering in an assisted living center near her home. I am hoping to be home by early August. I go through therapy every day and am on the road to recovery. I hope to be able to start school on time in August.

*Gloria Nan Dupree, President
C. D. Owen High School, Black Mountain, NC 28711*

Notes From The Secretary's Desk

Life is good. The summer has not been as hot as some I remember. I have been interviewing for jobs and have enjoyed meeting many new people – both in North Carolina and Tennessee. Teaching jobs are difficult to find and there are numerous applicants for every opening. I have also been informed that sometimes they are looking for younger teachers – it is not clear if they mean salary concerns or if they simply want someone who will be staying longer. Young at heart and experience should count for something – it made me wonder how many jobs I previously obtained due to youthful exuberance!! As for calculus, I had a great experience teaching a weeklong summer institute at UNC-Charlotte. It was great fun and as usual, I learned from my students. It was great to work with David Royster, the Summer Institute director. You may not know that David and his wife, Norma, are leaving NC to return to his hometown of Lexington, KY. David will be an Outreach

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Mathematician at the University of Kentucky. North Carolina's loss is Kentucky's gain! We are also going to miss David and Norma at the NCAAPMT Board of Director's meetings. *Deb Britt, Mars Hill, NC, dgb531@aol.com*

Please remember to renew your membership to receive the two yearly newsletters. You can send your \$5.00 check, payable to NCA² PMT, to Jeff Lucia, 718 Lansdowne Road, Charlotte, NC 28270. Email address is jeff.lucia@providenceday.org.

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North Carolina Association of Advanced Placement Mathematics Teachers
Minutes of the Meeting of the Board of Directors
Greensboro Day School, Greensboro, NC
June 20, 2009

Members Present: Gloria Dupree (presiding), Martha Ray, Geoffrey Lucia, Deborah Britt, Emogene Kernodle, Rhea Caldwell, Dan Teague, Norma Royster, David Royster Members Absent: Ray Jernigan, Sue Wall, Stephen Davis Guests: Patricia Morris

- 1) Gloria Dupree called the meeting to order at 10:00 a.m.
- 2) The minutes of the June 2008 meeting were officially approved.
- 3) Jeff Lucia presented a Treasurer's report. The balance in the treasury shows a net gain of \$86.75 with a balance on June 20, 2009 of \$4898.76. Mr. Lucia noted the importance of funding received through the UNC Charlotte Center for Mathematics, Science and Technology Education (CMSTE) and thanked David Royster for the continued support. This amounts to 25% of our newsletter costs. Everything is being done to hold costs down. Jeff also reported that the Statistics group will be publishing and mailing their own newsletter this summer. Deb Britt will contact Vicki Carter to see if the SC association is still functioning. The SC group has not been sending courtesy newsletters to NC board members, while we have been sending the NC one to SC board members.
- 4) Mr. Lucia also gave a membership report. The current membership is 351, which includes 101 whose membership expired 3/2009. Membership is down from 365 last year but is still stable. NC has 131 members, 217 members are from 37 other states and DC, and 2 members are from foreign countries.
- 5) A report from those attending (Deb Britt, Rhea Caldwell and Norma Royster) the AP Calculus reading was given. The 2009 reading was in Kansas City, MO. Overall, the reading went well. There was concern regarding students' lack of interpretation and justifications on the exam. A decision was made to focus more on this at the upcoming NCCTM meeting.

- 6) The North Carolina Council of Teachers of Mathematics Conference will be in Greensboro. Concern was expressed about the NCCTM conference being limited in size due to economic concerns. Rather than individual question scoring presentations, it was decided to focus on topics of student difficulty. Our session on the program will include Jeff Lucia – meanings and justifications, Dan Teague – calculator concerns and work presentation on calculator section, and Rhea Caldwell – graph interpretations, local and global issues.
- 7) Deb Britt announced the upcoming Calculus with Technology Conference within the T³ Conference in Atlanta. The T³ conference will run Friday, Saturday and Sunday (March 5 - 7) with the Calculus Conference on Saturday only and possibly Sunday.
- 8) An expression of deep appreciation was given to Norma and David Royster for their faithful service to the Board. Norma and David are leaving NC to move to KY. David will be doing mathematics outreach work across the state of KY.
- 9) Suggestions for new board members were made. Gloria will contact candidates as to their willingness to serve on the board.
- 10) There is ample content for the summer newsletter. Deb Britt offered to allow someone with more publication expertise to take over as editor. David Royster offered to set up a mock version of a newsletter in column format to see if perhaps this format might look better. It was mentioned that most mathematics publications use a single page format due to the many nuances with mathematical expressions.
- 11) Martha Ray was elected as President-elect. She will serve in this role for one year replacing David Royster, who had previously been elected to serve in this position.
- 12) June 19, 2010 will be the date of the next NCA²PMT board meeting unless this conflicts with AP Reading dates. The meeting will be held at Greensboro Day School with Patricia Morris hosting the event.

The meeting adjourned at 12:00 p.m.

Respectfully Submitted by Deborah G. Britt, Executive Secretary

NCA²PMT 2009 REPORT FROM THE TREASURER/MEMBERSHIP CHAIR

| | |
|---------------------------------|-----------|
| Balance as of 6/21/08 | 4812.01 |
| Deposits (memberships) | 1361.00 |
| Deposits (newsletter grants) | 425.14 |
| Deposit (postage reimbursement) | 11.40 |
| | |
| TOTAL | \$6609.55 |
| | |
| Newsletter (August 2008) | 1212.55 |
| Newsletter (February 2009) | 498.24 |
| Balance as of 6/20/09 | 4898.76 |
| | |
| TOTAL | \$6609.55 |
| | |
| TOTAL INCOME | 1797.54 |
| TOTAL EXPENSE | 1710.79 |
| | |
| NET GAIN 2008-2009 | \$86.75 |

The above gain for 2008-2009 represents our sixth consecutive year with an operating surplus, albeit a very small one. The following reasons can be cited:

1. We continue to receive a grant for each newsletter and mailing through Dr. David Royster from the UNC Charlotte Center for Mathematics, Science and Technology Education (CMSTE). These grants defray approximately one-fourth of our expenses. We express our sincere thanks to Dr. Royster and the CMSTE for their continued generous support. Without it we would show a deficit for the year.
2. We continue to hold down the cost of printing and mailing our semi-annual newsletter.

Current membership is 351, including 101 whose membership expired 3/2009 who will receive a reminder but will not get the summer newsletter unless they renew. This overall number is down from 365 last year at this time, and the number of paid up members is currently 250, as opposed to 266 last year. We continue to send courtesy copies of the newsletter to non-member authors of contributed articles. However, we will no longer be sending copies to the SCA²PMT board members, as I have received neither updated communication nor copies of their newsletter for several years. I'm not sure if they are even publishing a newsletter any longer. A breakdown of our membership is as follows:

| <u>North Carolina</u> | | <u>Other States</u> | | |
|-----------------------|----------------------|---------------------|-------------|---|
| Eastern - 36 | California | 33 | Connecticut | 2 |
| Central - 50 | Virginia | 24 | Hawaii | 2 |
| Western - 45 | New York | 19 | Kansas | 2 |
| | Florida | 13 | Kentucky | 2 |
| TOTAL - 131 | Georgia | 12 | Minnesota | 2 |
| | New Jersey | 11 | Missouri | 2 |
| | South Carolina | 10 | Oklahoma | 2 |
| | Texas | 10 | Wisconsin | 2 |
| | Maryland | 9 | Arizona | 1 |
| | Colorado | 8 | Delaware | 1 |
| | Michigan | 7 | Idaho | 1 |
| | Pennsylvania | 6 | Iowa | 1 |
| | Indiana | 5 | Maine | 1 |
| | Ohio | 5 | Mississippi | 1 |
| | Tennessee | 5 | Nevada | 1 |
| <u>Foreign</u> | Washington | 5 | New Mexico | 1 |
| Brazil - 1 | Massachusetts | 4 | Oregon | 1 |
| Turkey - 1 | Utah | 3 | Vermont | 1 |
| | Alabama | 2 | | |
| TOTAL - 2 | District of Columbia | 1 | | |

TOTAL OTHER STATES - 217 (NC plus 37 states & DC)

None from 12 states:

Alaska, Arkansas, Illinois, Louisiana, Montana, Nebraska, New Hampshire, North Dakota, Rhode Island, South Dakota, West Virginia, Wyoming

Submitted by Jeff Lucia, Treasurer/Membership Chair, June 20, 2009

NOTES FROM THE AP READING – Kansas City 2009

AP Reading

Deborah Britt - Ohio University, Athens, OH

The Reading for me was lots of fun since it seemed like every time I turned around, there were readers from many of my former years. The number of readers was a cut from prior years. There were fewer new readers (approximately 100) and more experienced readers, even with a 4% exam increase from last year. Our reading day was from 8 AM until 5 PM (an increase from prior years that used to begin at 8:30). We had a larger opening session room and the computer room for checking email moved locations. There were three workshops and a professional night (all discussed later in this issue). A microphone was included in the social lounge, making announcements easier to hear. I was assigned to the alternate exam room and loved grading so many problems over the course of the seven days (at last count, we did 11 different problems). More students took the alternate exam (a 40% increase) and it was believed that this was due to swine flu (H1N1 virus) school closures during the regular exam time. The T-shirts we purchased were black with white lettering and featured a calculus Sudoku puzzle on the back.

Chief Reader Michael Boardman had a new bearded look that kept most of us from immediately recognizing him – which he seemed to enjoy. He stated there were 304,490 exams to grade, 1,826,940 questions and 16,442,460 possible points to be awarded. From the first exam in 1955, there have been 4,200,000 exams administered. There were 835 readers representing 49 states with 48% being

high school teachers and 52% college. One new term emerged for our calculus grading terminology – bankable. Bankable meant that once the point was awarded, it could not be taken away – no matter what contradictory or bad work followed. Not all points are bankable on the grading rubrics.

Opening Session of 2009 Advanced Placement Calculus Reading Carol Lowe - North Surry High School, Mount Airy, NC

We arrived on Monday, June 8 from 49 states, the District of Columbia as well as other countries. Bright and early Tuesday morning—8 am to be exact—we were greeted by Chief Reader, Michael Boardman, from Pacific University—a veteran AP Reader of 16 years. He welcomed the 835 participants, 105 of whom were new readers (affectionately referred to as "acorns"), and informed us that we would be reading 304,490 AP Calculus exams. Yes, that would be a total of 1,826,940 free response questions with the possibility of earning 16,442,460 points. To impress upon us the importance of our being here as readers, he pointed out that without us, it would take him almost 10 years to read the exams alone, and that certainly would not get scores back to students in a timely manner!

The remainder of our opening session was spent giving recognition to leaders of the reading, learning about special events, workshops and meetings available to us throughout the week, and reviewing general guidelines for grading free response questions. Upon entering the meeting room, we each received a jargon sheet, explaining terminology familiar to veteran readers and intimidating to the acorns. So just what does it all mean—bald answers? copy errors? inoculation? table scrapping? We were reminded of important guidelines including:

- Keep student books in order
- Student books stay in reading rooms
- Do not write in student books
- **Keep student books in order!**
- NEVER mark in student books
- Do not read erased or marked out work
- **KEEP STUDENT BOOKS IN ORDER!!!**

We discussed decimal presentation, appropriate calculator use, table partners, table leaders, and question leaders. In closing, Mike reminded us that it really all boils down to one thing—consistency. It is imperative that student free response questions are graded consistently and we were encouraged to work for consistency—not speed. He wished us the best, introduced us to the Question Leader for problem 1, and the work began.

Tales of a New Table Leader Nicole Lang - North Hennepin Community College, Brooklyn Park, MN

Being a rookie again is, well, ... exactly the way it sounds. In this case, being a new table leader at the AP Calculus Reading was a somewhat surreal experience. I was new to some parts of the job, but quite familiar with most facets. Therefore, I was new without *really* being new. In many ways, my first year as a TL resembled my experience as a student teacher. I had a pretty good idea of what was expected of me, but the reality of the job was overwhelming at first. It would have been especially daunting had I not been paired with such a wonderful, experienced TL partner.

On Backreading: The learning curve for the Art of Backreading was fairly steep (at least for me). How often do I take a contested score back to a reader? Which are issues important enough to discuss the scoring standard with a reader? Which issues are subtleties that are rare enough that it's not worth the extra time and energy to walk to the other side of the room? How long should this take? I was a whole day into the reading before I even knew what questions to ask!

With more readers per room than in previous years, those stacks of backreading built up really fast, and I felt really slow! I gradually learned how to prioritize and picked up other tips and tricks to help me be efficient, but the first two days were pretty rough. There were many times I wished for someone to backread me so I could get some external sense of how I was doing. For that reason, I made a point of talking to all the readers that I backread – even if they had no contested books. Who doesn't like to know if they're doing a good job?

On Answering those Tricky Questions: After the first hour or two on the first day, answering the questions became much easier. This was probably my favorite part of the job. Readers in my room brought up some tough questions and issues, and it both challenged and deepened my understanding of the grading standards and philosophies we adopt for the week.

On working with an outstanding group of my peers: My fellow TL's and all of the readers in my room were (as always) wonderful people with whom to work. More experienced TL's were quick to sympathize and share their own backreading strategies. My room was full of dedicated professionals who did a great job learning and applying the standards to the questions we read, which made my job much easier. The camaraderie I experienced as a TL was the same as I had experienced as a reader.

Overall, my first experience as a table leader was positive. There were challenges, of course, but there were also some great rewards. I made new friends and learned new ideas, skills, and strategies. It was an honor to be chosen as a table leader, and I am simultaneously proud to have been selected and humbled by the experience. I hope to be invited back as a TL so that I can continue to improve and learn.

Test Development Committee Night

Julie Harrison - Eagles Landing High School, Mc Donough, GA

The AP Calculus Test Development Committee hosted an evening session at the AP Reading where committee members provide information about the AP Calculus Tests as well as offer a chance for participants to ask questions of the committee. The committee consists of Chair Stephen Davis of Davidson College in North Carolina, Robert Arrigo of Scarsdale High School in New York, Tom Becvar of St. Louis University High in Missouri, Ruth Dover of Illinois Math and Science Academy, James Epperson of University of Texas-Arlington, Kathleen Goto of Iolani High School in Hawaii, and Tara Smith of University of Cincinnati, as well as ex officio members Chief Reader Michael Boardman of Pacific University in Oregon and ETS Consultants Fred Kluempfen and Craig Wright. It was noted at the session that this committee generally has three meetings a year each lasting for 3 days plus as many virtual meetings as they find necessary in order to fulfill their responsibilities as indicated below. It was also noted that members typically serve on the committee for 4 years but with a yearly "renewal."

The multiple responsibilities of the Test Development Committee were addressed and explained during the session. One of the main responsibilities of the committee, of course, involves planning, developing, and approving each exam. The group participates in the multiple-choice component as well as the free response part of the exams. The committee also participates in the Reading each year, which the members view as very valuable in their quest to craft the AP exam. Another important responsibility of the committee is the participation in statistical reflection on previous years' exams. Other important aspects include serving in an advisory capacity on the course audit and to the Chief Reader on reading issues as well as participating in outreach events such as this session and sessions at the AP Annual Conference. They also participate as a group in the Special Focus issues that the College Board puts together.

Of note at the session was the fact that the committee also reviews and updates the Course Description. Committee Chairman Davis noted that a new Course Description is coming for the 2009-2010 school year in which a few sample questions may be replaced with new ones as well as an inclusion of a revised statement on the need for students to show their calculus work. (This new course description is now available at www.apcentral.collegeboard.com.) At this point the Chief Reader, Michael Boardman, interjected and informed us that he asked the committee to include a "stronger statement" on the necessity for students to show their calculus work on the exam. He emphasized that the goal was to "see student thinking" on the free response questions and for the free response questions to validate the student's multiple-choice score.

A bit of time was spent making sure people were familiar with the setup of the exam. On each exam there is a multiple-choice part and free response part, both of which contain a calculator and non-calculator section. There is also an AB exam and a BC exam, and the exams share questions called "common problems," which are used to find the AB subscore for BC exam takers. The "Operational" exam is administered in North, Central, and South America, including Alaska and Hawaii. The "Form B" exam is given overseas but given on the same day as the operational exam. The "Form A" (or alternate) exam is given later to accommodate causes that require testing at a later date. This year more kids took this alternate exam, possibly due to issues with H1N1 (swine flu), but an additional room at the reading was devoted to this Form A exam instead of the Operational exam due to this increase.

A portion of the session was devoted to discussion of multiple choice questions. Committee Chairman Davis pointed out that they make some of the MC questions and the College Board contracts out for some to be made (usually by former committee members). Several issues were stressed such as the need for "clear, idiom-free phrasing" in the questions as well as questions that focus on a specific topic to illuminate whether or not the student knows that specific concept. He also emphasized the need for choices that are "appropriate distracters." This concept included needing just one correct/reasonable answer, avoiding "none of the above" as a choice, including "distracter buddies" such as sine and cosine, and the need for choices that illuminate calculus knowledge. I gathered that when discussing multiple choice questions and their choices the committee takes into consideration whether or not a student might make an algebra mistake in the process of attempting a question. If an answer choice is there for a particular common algebra mistake then that option might likely be eliminated as a distracter. It was noted by Committee Chairman Davis that the multiple choice questions were "very directed" and addressed particular skills partly because they don't want students skipping questions if they were too long or involved, since that does not tell what a student knows about calculus.

When committee members write multiple choice questions they are to keep several things in mind. They must incorporate a mixture of calculator, non-calculator, and "calculator-neutral" questions as well as incorporate a variety of function types (meaning algebraic, exponential, logarithmic, and trigonometric). Members must also incorporate the rule of 4 that encompasses the symbolic, graphical, tabular, and verbal aspects as well as be mindful of how long the question takes, the concept it addresses, and its level of difficulty.

Another big responsibility of the committee is developing free response questions for each year's exam. Between the AB and BC exam, 9 questions must be made (due to 3 common questions and 3 separate for each exam) for all 3 forms of the exam (operational, Form B, and Form A), which means composing 27 questions per year. Free response questions are made well in advance and the initial rendering of the question may be transformed to almost unrecognizable - except for the core idea - by the time it hits the exam for students to see. Questions are typically developed and exam constructed two years in advance - the committee is currently working on the free responses for the 2011 exam. Contrary to the belief held by some people, the committee does not take what students did poorly on and try to put it on the next year's exam. Committee Chairman Davis emphasized that the committee "does not approach the exam or questions with a particular agenda ahead of time." One of the goals in the making of free response questions is to make the wording as clear and succinct as possible. It was communicated that one of the big challenges in making questions in the calculator portion is to not advantage or disadvantage any particular calculator type (CAS or not). When making the questions the committee discusses if the questions contains 9 points or perhaps more or less than 9 and how to change it to reflect a possible valid 9 points. Once the questions actually are administered the committee advises on the potential rubric but it is the responsibility of the Chief Reader to make the final decision on the rubric. The goal on any of these rubrics is to "reward good communication on the exam," which corresponds to the increased emphasis in the new course description informing teachers to have students show their work.

During the question time, there were several good questions asked with informative answers that followed. Someone asked if there would be potential syllabus changes in the future, and the response was that there is a Review Committee conducting a review of multiple AP courses and the findings from that would be analyzed before a decision was made on any syllabus changes. Another question addressed the Practice Test made available through the Audit process and whether or not a new one would be made available. Committee Chairman Davis noted that the Practice test was not made by the Development Committee and was not sure if/when another test would be made available through the audit. Yet another question came from a person who asked how the committee decides how long a free response question takes since the students have essentially 15 minutes per free response question. Of course judging this is difficult, but the committee relies on the multiple perspectives of the members on the committee and the fact that they rework the questions essentially each time that they meet and discuss the questions. Committee Chairman Davis expressed that each time that they meet and work the questions it is like seeing them for the first time due to a variety of factors. He explained that if they feel that a question is "too long" then they get rid of it. In fact, I believe that the sophisticated process by which a rubric is finalized (from initial inception in the thoughts of the development committee to the analyzing of student responses by question leaders and others before the readers actually arrive) and applied that questions are truly able to reflect whether or not a student understands calculus well.

Another interesting question addressed what we should expect on the exam in the future. It was conveyed that generally we should expect several things such as two problems in context and for these situations to represent those that are "relevant to kids." We should also not expect "sameness from year to year" and should also continue to look for an emphasis on the Rule of Four. We should also potentially expect area/volume free responses to appear in the non-calculator section again, since members of the committee "are not happy with it in the calculator section" due to the availability of CAS systems that will tell you how to actually setup the problems. With that in mind, these questions may appear in the non-calculator portions or perhaps be written in such a way as to not give the students with that capability an advantage. Beyond these thoughts it is impossible to predict what might appear, which I believe emphasizes our job as calculus teachers to make sure students thoroughly learn and understand calculus and are able to communicate their knowledge clearly and with adequate support.

Professional Development Night Talk by Larry Riddle, Former Chief Reader Kay Westbrook - Allen High School, Allen, TX

I had the pleasure at the AP Calculus Reading of hearing Larry Riddle speak. Larry's background as a former Chief Reader and current Professor of Mathematics at Agnes Scott Women's College in Atlanta, Georgia gave him the motivation to research and speak about pioneering women in Mathematics. As a female math teacher, I have always had a curiosity about the contributions of others in the field. Most of the discussion centered around Sophie Germain, who was born April 1, 1776. Though discouraged by her parents to study math, she persevered to study differential equations and the work of Gauss and Fermat. She made considerable progress in proving Fermat's Last Theorem, which was finally proved in 1995 by Andrew Wiles. Her initial writings were published under a male name in order for her works to be taken more seriously within the mathematical community. As with many mathematicians and scientists throughout history, Sophie was not honored for her contributions until after her death. There is currently a street named after her in France. Surprising to me were the mathematical contributions made by Florence Nightingale. I had heard of her contributions as a nurse during war time. I did not realize what a difference she made in preventing the spread of disease through her

statistical tracking of admissions records and deaths. Her research led to major changes in sanitation and data collecting and recording. The first PhD in Mathematics awarded to a female was awarded to Sonja Kovalevsky.

I only wish I had heard of some of the more recent contributions of women to the field of Mathematics. The talk ended on a light note. Larry showed a clip from "The Simpsons." In this episode, Lisa Simpson dresses as a boy in order to be able to fit into the advanced math class at her school. She surprises everyone by winning the most outstanding math student award. This is the link to Larry Riddle's web site on women in math: <http://www.agnesscott.edu/Lriddle/women/women.htm>

AP Forum

Norma Royster - Charlotte, NC

The College Board AP Forum met on June 10, 2009 in the theater at the reading site. Statistics were given on the growth of the AP program and comparisons between non-AP and AP students, with SAT and SES used as controls to do the comparisons. The study appears at www.collegeboard.org. There will be some new submissions to the audit due to changes in some courses. Thirteen courses (not calculus) will be under these new changes. For 2009 approximately 3 - 4 courses will have minor to substantial changes to the course of study. There was some discussion about site changes two years from now. No definite changes yet. Readers were encouraged to give suggestions on how to improve the site conditions. For example: have a check box for choice of hotel on the form returned concerning housing.

Workshop Night Talk by Tom Dick

T. Michael Brown - The Prairie School, Racine, WI

How should technology be used in the teaching of mathematics? If a \$4.99 calculator can divide two numbers correct to eight digits, should students be taught the long division algorithm? Should students be taught to factor polynomials? Should they be taught to integrate a function using trigonometric substitution? Or take the converse of one of these questions. Should a student be allowed to use technology to perform any or all of these tasks? And, if so, when? These questions continue to vex mathematics teachers, and the rapid advancement of technology means that the questions are being considered in a changing scenario.

Dr. Thomas Dick, in a presentation to readers at the 2009 AP Calculus reading in Kansas City, suggested an interesting way to look at this issue through what he called a "different lens." Dr. Dick suggested the need to view technology as more than simply a "computational chore performer." We instead need to view technology as an environment for illustrating mathematical ideas.

This approach to the use of technology in teaching mathematics is based upon what Dr. Dick called the "Action / Consequence / Reflection Principle." Technology should be used to afford the student an opportunity to take an action on a mathematical object and observe the consequence. In a simple example, two points were shown on a coordinate plane. Moving point Z caused point W to also move based upon some rule that connects the points. The student moves Z and observes the consequence, the resulting movement in W. The student is then asked to reflect upon the resulting movement and draw conclusions about the rule connecting W to Z.

What is the role of the teacher in this interaction? The teacher first needs to find these "sense-making" activities in which students can interact with a mathematical system and observe the consequences immediately. They must be "engaged in mathematics at the level of the screen," not simply interacting with the machine. The teacher must then "seal the deal" by asking good questions that lead the student to actively learn through reflection.

Dr. Dick also presented some examples for calculus teaching (Riemann sums, the relationship between f and f' , Taylor polynomials, etc). More important than the precise subject matter, though, he left the listeners with a way of rethinking perplexing questions about teaching and technology.

Workshop Night Talk by James Epperson

Stephen Bowling - Southwest Baptist University, Bolivar, MO

One of the challenges with going to a buffet is that there are so many good things to choose from. When choosing what to eat, a person should take into consideration what is good for him right then, what will be good for him in the long run, and also what he would enjoy eating. At the AP Calculus grading in Kansas City, Missouri, this summer, there were many opportunities to "dine" on various presentations during the evenings, and these had had to be balanced with the opportunities of seeing the sights of Kansas City and relaxing and interacting with high school and college mathematics teachers from around the country and the world.

One of the presentations available to "dine on" during an evening was a presentation by Dr. James Epperson on "Balancing Calculus, Technology, and Critical Thinking in the Classroom." Dr. Epperson, who is an Associate Professor of Mathematics at the University of Texas at Arlington, gave his presentation using a Geometer's Sketchpad file. Dr. Epperson also serves on the Test Development Committee.

Dr. Epperson mentioned early in his presentation that there are many strategies that can be used to solve problems. Unfortunately, many freshmen students seem to know only one strategy, trial and error. He got the audience to engage in some critical thinking by having us contrast the definition of $\lim_{x \rightarrow a} f(x) = L$ with an intuitive idea that I and others have used in order to try to make the concept of the limit simpler for the students to understand, the idea that $f(x)$ gets closer to L as x approaches a . Dr. Epperson gave us illustrations that showed that this intuitive idea is neither stronger than nor weaker than the actual definition of a limit, so that these two concepts are not equivalent.

Throughout his presentation, Dr. Epperson had many good interactive illustrations on various sheets of his Geometer's Sketchpad file that Calculus teachers could find useful in helping students understand various Calculus concepts, including the concept of the limit. Many of these sheets in his Sketchpad file included graphs that could be adjusted by the user and tables of values that could be expanded or adjusted by the user. In particular, one sheet allowed the instructor to show visually how for a specific function, adjusting ϵ can affect the δ needed to insure that $f(x)$ is within ϵ of L .

In regards to using technology, Dr. Epperson warned the audience to be careful of putting too many things on a single sheet or application, especially if students are not yet used to a type of technology. Having too many bells and whistles on an application can obscure or distract from the general point that is being illustrated. Dr. Epperson also illustrated that one must be careful to not depend too much on technology by showing that Geometer's Sketchpad does not give a good graph of $y = \sin\left(\frac{2\pi}{x}\right)$.

There was another valuable technique/reminder that Dr. Epperson showed us: he gave more than one good example of asking students to solve a problem in more than one way. This is something that I will want to try to implement more in the future.

In short, Dr. Epperson gave a talk that had the audience engage in some critical thinking, illustrated some practical ways technology may be used to teach Calculus concepts while giving a couple of warnings of things to beware of when using technology, and gave examples of asking students to find more than one way to solve a problem.

Dr. Epperson has a website available at <http://www2.uta.edu/math/epperson/>, and he may be reached by e-mail at epperson@uta.edu.

Reading the Alternate Exam Roger Smith - Davis College, Toledo, OH

This was the year for me to be selected to read the Alternate Exam of the Advanced Placement Calculus Test. In 2009, a special year occurred. With the threat of a major flu epidemic, many schools were not able to administer the regular test on the specified day, so over 40,000 students needed to take the Alternate Test. There were also some students that had to take an alternate Alternate Test. This produced three rooms of readers with each room having 16 Readers and 2 Table Leaders. Over fifty professionals were assigned to do this special test. The first task was for each reader to sign a release form since the test was a secure one. That meant that the test will not be released and no comments could be made outside the reading room. Also, each room was secured so that the test could not be seen by anyone other than the designated readers.

We started with grading the problems on the regular exam. Part way through the week we started grading the Alternate test problems. This meant that our readers had to attend additional briefings to have the rubrics explained. With such a small group, these sessions were much shorter than the usual briefings. When all the tests were read, our group had been briefed on eleven questions in total. The other readers had at most five briefings.

Our table leaders were well experienced in that task and extremely helpful. They represented some of the best of that group. The individual readers also had many years of experience in the readings, with an average of 9 years of attendance. Much help was available to each reader with so much talent and knowledge available to each other.

The test was a fair one and each problem covered an important topic in the Calculus sequence. With the Alternate Test, we had to grade many problems and the flow seemed to go well. Towards the end of the reading, extra boxes of tests, some secure and others released, needed to be graded. These were brought to the Alternate rooms, since the briefings had been given and the readers could grade so many different problems. Sometimes, a table would split a packet of tests. And we trained to grade both the AB and BC

forms. When the week was over, a good deal of satisfaction was felt by each reader since so much had been accomplished in many different areas. Each reader hoped that he/she would be selected to do this same task next year.

Grading the FORM B Test

Beverly McCarthy - Health Careers High School, San Antonio, TX

Editor's Note: Form B readers are usually very experienced readers. Prior experiences on a multitude of problem types allow them to grasp quickly the concepts of new rubrics and to be able to switch problems whenever needed. Versatility and adaptability with demonstrated speed and accuracy are desired qualities in the Form B and Alternate exam rooms.

The **Form B** test is the AP test administered to students living overseas. The test is different from the operational exam in order to minimize problems with students taking the exams in different time zones. The questions are posted on AP Central. This year, there were two rooms that graded the Form B exams. Both rooms shared the responsibility for the AB and BC tests under the leadership of Deanna Caveny. The grading went smoothly, allowing the readers in these rooms to grade several problems on the operational exam as well. What follows are comments based on my observations. These observations may not prove accurate when all the data is analyzed.

Form B AB 2 / BC 2 began by defining a function for the rate of change of the distance between the edge of a road and the edge of the water at a beach. Parts (b) and (c) seemed to give the students the most trouble. Part (b) asked for the interpretation of the expression $f'(4) = 1.007$ using correct units. Since f was a rate, f' was the rate of change of the rate of change, something that is difficult to express accurately, even for native English speakers. Given that a lot of the students taking this test learned English as a second language, I was impressed with their ability to explain this concept accurately – probably better than my own students would have done. Part (c) asked when the distance between the water and the edge of the road was decreasing most rapidly – a global minimum. Some students looked for a maximum instead and some relied purely on the graph to answer the question.

Form B AB 4 was sort of an inverse of the operational AB 4, using $y = \sqrt{x}$ and $y = x/2$, but, unlike the operational exam, it included a solid of rotation. The students had the most difficulty with the algebra involved with squaring the binomial $(\sqrt{x} - x/2)$ correctly and then finding the anti-derivative and substituting the limits of integration. They seemed to have a good grasp of the washer method for volumes with the region rotated about a line other than the x-axis.

Form B AB 6 was a particle motion problem based on tabular data. The distinction between displacement and distance traveled gave some students difficulty. This is similar to the problems students had when explaining the meaning of the integral representing Caren's distance traveled on the operational AB 1. Part (d), requiring the use of initial position and displacement to explain why position at $t = 8$ had to be greater than 30 meters, presented the biggest challenge.

Form B BC 1 was the "double bubble" problem that asked about area, volume, and perimeter. (Editor's note: Double bubble means that 5 of the 9 points count toward the AB subscore and graders bubble those 5 points on one sheet and the last 4 points on a second sheet.) This problem seemed to be easy for students and served as a good bridge between the multiple choice and free response sections.

Form B BC 4 involved polar equations. Part (a) asked the students to write an integral for an area, and the students did well. Most errors here occurred in the limits of integration (or totally forgetting how to find polar area). Part (b) was the most interesting part to grade. Most students did not use the method outlined on the standard, but multiplied out $x = \cos\theta(1 - 2\cos\theta)$ and $y = \sin\theta(1 - 2\cos\theta)$ before differentiating. This led to what seemed to be an infinite variety of trig expressions for the answers, which made the reading interesting but slow.

In addition to grading the five form B questions listed above, our room graded four operational questions. The variety and change of pace, as well as having intelligent and amusing colleagues, made working in the Form B room a memorable experience.

Some Fundamental Difference Thoughts

Lin Mc Mullin – posted Tuesday, June 16 on <http://linmc-thelimit.blogspot.com/>

I returned today from the AP Calculus reading. Seeing all those exams (304,490 more or less) is great, and seeing all the different solutions is quite an experience. I would like to discuss several parts of the problems that all relate to the Fundamental Theorem of Calculus (FTC).

Let's start with 2009 AB 2 / BC 2 part (c). Students were given $R(t) = 1380t^2 - 675t^3$ and $w'(t) = (2-t)R(t)$ for $0 \leq t \leq 2$ hours. They were asked to find $w(2) - w(1)$. You could almost hear them thinking "First I have to find $w(t)$, then evaluate it at two points and subtract. I can do that." And in fact this works. After far too many lines of computation by hand (omitted here), many students (who were good at this sort of thing) came up with the correct answer:

$$w(t) = 135t^5 - \frac{1365}{2}t^4 + 920t$$

$$w(2) = 760$$

$$w(1) = 372.5$$

$$w(2) - w(1) = 387.5$$

But wait doesn't $w(2) - w(1) = \int_1^2 w'(t)dt$, and can't you calculate that on your calculator? Do you really need to know $w(t)$?

Then let's look at 2009 AB 3 part (b). Students were asked to explain the meaning of $\int_{25}^{30} 6\sqrt{x}dx$ using correct units in the context of the problem. The integrand was given as the cost to produce a portion of cable that is x meters from the beginning of the cable in dollars per meter. Explaining the meaning of a definite integral has been asked often on the AP Calculus Exams. Three things are required in the explanation: 1. what the integral represents; 2. correct units; 3. an accounting for the limits of integration. There were many different approaches, with many convoluted sentences. The most common mistake may have been forgetting the units (dollars) or giving the wrong units (dollars per meter). Answers like, "The cost in dollars of producing the last 5 meters of a 30 meter cable," and "The cost in dollars of producing the part of a cable between 25 and 30 meters from the end" earned the point. Students have trouble with all that, of course. So my suggestion is to think of the FTC:

$$\text{Let } 6\sqrt{x} = f'(x). \text{ Then } \int_{25}^{30} f'(x)dx = f(30) - f(25).$$

Now it should be clear that the simplest, most straightforward answer is "The definite integral represents the difference in dollars in the cost of producing a cable of length 30 meters and a cable of length 25 meters." And it works even if we don't know or can't find $f(x)$.

In 2009 AB 1 / BC 1 part (b) students were given a velocity expression $v(t)$ in miles per minute (a graph, actually) and asked to explain the meaning of $\int_0^{12} |v(t)|dt$. The absolute value of velocity is speed and the integral of speed is the distance traveled. If we let $p(t)$ be the distance traveled, then $\int_0^{12} |v(t)|dt = p(12) - p(0)$. So the meaning is easily seen to be the "distance traveled in miles from $t = 0$ to $t = 12$."

Finally, in 2009 AB 6 a few students tried to justify an absolute maximum of a function f given the graph and equation of $f'(x)$ on the closed interval $[-4, 4]$ as explained next. Alas, few, if any, were successful with this approach, but I liked the idea. The absolute maximum occurred at a point where $x = M$ (the actual value of M was given). They could have used the Candidates' Test and reasoned this way:

$$\int_{-4}^M f'(x)dx = f(M) - f(-4) > 0$$

$$f(M) > f(-4)$$

$$\int_M^4 f'(x)dx = f(4) - f(M) < 0$$

$$f(M) > f(4)$$

The inequalities in the first and third lines above are true because $f'(x) \geq 0$ for $-4 \leq x \leq M$ and $f'(x) \leq 0$ for $M \leq x \leq 4$ as seen from the graph. Perhaps the reason I liked this is that I never quite thought of using the FTC in any of these ways until I realized what

students were doing. Even if you don't know or don't want to bother computing the antiderivatives, thinking of them this way may help your students better understand the FTC as something more than a way to evaluate definite integrals.

****** Please visit the AP Central Website for copies of the 2009 Operational AP Exam questions ******

AB 1 / BC 1 – Caren and Larry Go Back to School

P. Rosemary Peeler - Convent of the Sacred Heart, New York, NY

In this problem, students had to deal with two representations of velocity, one graphical, the other algebraic. They were asked to interpret acceleration, distance and displacement.

Part (a) - 2 points: Students were given one point for the value of acceleration and one point for units. A bald answer for acceleration was not accepted. Students had to produce a correct difference quotient, a correct difference or a correct quotient; they could also work with the idea of acceleration as the slope of the velocity graph. If they produced a correct equation of the velocity segment and used its slope, this was also acceptable. The point for units was given only if it was attached to a number.

Part (b) - 2 points: One point was awarded for the correct interpretation of the integral and the other was earned for computation of the integral. To earn the first point, students had to appeal to distance, to the time interval and to units (although units could be attached to the evaluation of the integral). To earn the second point, there had to be evidence of how the value of the integral was determined; readers looked for formulas or for numbers on the graph in the student's answer booklet.

Common errors: Many students lost the first point by leaving out one of the required elements (usually the time interval), by confusing distance with displacement, or by thinking that the units were in miles/minute. The second point was lost by students who did not back up their computation or who simply arrived at a wrong number. A significant number of students did not read the velocity axis correctly and had an answer of 18 miles rather than 1.8.

Part (c) - 2 points: The first point was awarded for declaring $t = 2$ as the time when Caren turned back. The second point had to reference the fact that the velocity changed sign or **became** negative as t passed through 2.

Common errors: Although most students correctly cited $t = 2$, some identified $t = 1$ as the turning point since it was the point when the velocity graph changed direction. Many students lost the second point for incomplete or inaccurate statements (e.g., at $t = 2$, $v < 0$; when $v = 0$, a particle changes direction; the area below the time axis equals the area above the axis).

Part (d) - 3 points: To earn the first two points, students had to compute the integral for Larry's distance. To earn the final point, they had to compute Caren's distance from school and draw a conclusion. If students computed an incorrect total distance in part (b), they were expected to deduct 0.4 from that number to get credit in part (d). The exception was for students who computed a distance of 1.4 in part (b); these students could use either 1.0 or 1.4, the correct distance, in part (d).

Common errors: Most students earned the first two points, although several tried a numerical approach to evaluating the integral. For students who computed Caren's distance as 1.8 in part (b) the most common error was in computing her distance from school as 1.6 and concluding that Caren and Larry lived the same distance from school. The other most common error was in using the part (b) distance rather than appealing to displacement or to the distance traveled from $t = 5$ to $t = 12$.

AB 2 / BC 2 – The Rock Concert Problem

Ryan Pietropaolo - North Carolina School of Science and Mathematics, Durham, NC

AB 2 / BC 2 is a multi-faceted problem that originates with a given function that measures the rate at which people enter an auditorium for a rock concert, hence it became known as the "Rock Concert Problem." The problem is split into four parts and is calculator active. The main concepts deal with the Fundamental Theorem of Calculus, using derivatives to find extrema, and average value (mean).

Part (a) – 2 points: The philosophy for earning these points focuses on the student being able to recognize the total change as the definite integral $\int_0^2 R(t)dt$ (1st point) and being able to evaluate the integral to obtain the correct answer (2nd point). Students earned the first point (concept) point by showing any of the following:

$$\int_0^2 R(t) dt$$

$\int R(t) dt$ and evidence of evaluation at $t = 2$

Correct antiderivative $\frac{1380}{3}t^2 - \frac{675}{4}t^4$ and evidence of evaluation at $t = 2$

Evidence of evaluation is determined by intent to evaluate at $t = 2$, either by direct substitution into the antiderivative of $R(t)$ or with an expression such as $\left. \frac{1380}{3}t^2 - \frac{675}{4}t^4 \right|_{t=2}$. It was not necessary to evaluate at $t = 0$, since the antiderivative at that point yields zero.

If students copied the problem incorrectly they could not earn the first point but were eligible for the second point. They earned the second point (answer point) for any of the following responses:

1. 980; if the first point is earned, intermediate work need not be correct
2. A copy error consistent with the answer to the student's definite integral
3. $\int R(t) dt = 980$; student does not earn first point if no evidence of evaluation is given

Part (b) – 3 points: Students were asked to find the time at which the rate of people entering the auditorium is a maximum. The philosophy for earning these points focuses on recognizing the concept of finding a maximum value (1st point), solving the appropriate equation (2nd point), and justifying that the value obtained for t does indeed yield the maximum (3rd point). Students earned the concept point for any of the following responses:

1. Considers the equation $R'(t) = 0$
2. Considers a sign change of $R'(t)$
3. A verbal description such as "the derivative of $R(t)$ changes from positive to negative"

Students earned the point for solving for critical numbers by identifying $t = \frac{184}{135} = 1.362$ or 1.363 as a candidate for the time when the maximum occurs. (Note: if the student has the correct answer but rounds or truncates to two or one decimal place, he or she loses this point but will not be penalized for future rounding errors on this problem.) Finally, students were eligible to earn the justification point if the first two points were earned and the student made a global argument (appealing to the interval $0 \leq t \leq 2$). Any of the following responses were acceptable:

1. Endpoint analysis: $R(0) = 0$ $R\left(\frac{184}{135}\right) = 854.527\dots$ $R(2) = 120$
2. $R'(t)$ changes from positive to negative at $t = 1.363$ and $R'(t) > 0$ for $0 \leq t < 1.363$ and $R'(t) < 0$ for $1.363 < t \leq 2$
3. Verbal explanation: $t = 1.363$ is the only time on the interval $[0, 2]$ where $R'(t) = 0$ and $R'(t)$ changes from positive to negative.

Note: Local arguments are not sufficient, i.e. " $R'(t)$ changes from positive to negative at $t = 1.363$ " or "the slope of R changes from positive to negative at $t = 1.363$."

Part (c) - 2 points: In part (c) students are given the derivative of a function $w(t)$ which represents the sum of all wait times for everyone that attends the concert from the moment each person enters the auditorium until the concert begins. The philosophy for earning these points focuses on recognizing the Fundamental Theorem of Calculus, i.e., that $w(2) - w(1) = \int_1^2 w'(t) dt$ (1st point) and correct evaluation of this definite integral (2nd point). Students earned the concept point for any of the following responses:

1. $\int_1^2 w'(t) dt$
2. $\int w'(t) dt$ and evidence of evaluation of the antiderivative of $w'(t)$ from $t = 1$ to $t = 2$
3. $w(t) = 920t^3 - 682.5t^4 + 135t^5 + C$ with supporting work and evidence of evaluation from $t = 1$ to $t = 2$.

Students earn the answer point for having 387.5 anywhere on the paper as long as the first point is earned. The following are scenarios where the second point is earned but not the first point:

1. A copy error in the integrand (one numerical mistake or missing parentheses) with answer consistent with the integral
2. Correct integrand in an indefinite integral with answer 387.5

3. Correct antiderivative of $w'(t)$ with no supporting work but with evidence of evaluation from $t = 1$ to $t = 2$ and a correct answer

Note: Copy errors (including missing parentheses) in the integrand are deducted from the first point, but the students are still eligible for the second point. Computational errors come off the second point.

Part (d) – 2 points: The philosophy for these points focuses on recognition that the total wait time is represented by the expression $w(2) - w(0) = \int_0^2 w'(t)dt$ (1st point) and that the average wait time per person can be calculated by dividing the total wait time by the total number of people (2nd point). Students earned the concept point for any of the following responses:

1. $\int_0^2 w'(t)dt$
2. Providing the correct numerical value (760) if calculated in part (c)
3. Explicit expression for $w(t)$ with $w(2) = 760$

The answer point was earned for any of the following responses:

1. $\frac{760}{980} = .775$ or $.776$
2. $\frac{760}{\text{answer from (a)}}$ (may import an incorrect answer from part (a))

Note: Students earn the second point but not the first point when they correctly calculate $\frac{1}{980} \int_a^2 w'(t)dt$ for any $a \in (0,2)$.

AB 3 – The Cable Problem Rhea Caldwell - Providence Day School, Charlotte, NC

The third problem of the calculator active free response problems dealt with the Mighty Cable Company's manufacturing of cable selling for \$120 per meter. Students were required to determine profit for a 25-meter cable, find the length of cable which would maximize profit, write a general expression to determine profit for any length, k , of cable, and also to explain the meaning of an integral expression in the context of the expression. Units, understanding of the Fundamental Theorem of Calculus, and explicit language were crucial to acquiring all points on this problem.

Part (a): Profit was defined for students as the difference between cost and revenue, so to receive the first of the two points for this part of the problem students had to build an entire profit relationship using calculus to determine the revenue of a cable x meters long at a rate of $6\sqrt{x}$ dollars per meter. Thus, the evaluation of $P(x) = 120(25) - \int_0^{25} 6\sqrt{x} dx$ gave the desired solution of \$2500. No points were awarded if a student used an indefinite integral. Students who ignored the units for the revenue relationship received no points for writing $120(25) - 6\sqrt{25} = \$2970$. Many students did not recognize $6\sqrt{x}$ dollars per meter as a rate of change and that integrating a rate of change would give the difference in the value of the cost of the cable, by the FTC.

Part (b): Students were given one point for the correct explanation of the integral expression $\int_{25}^{30} 6\sqrt{x} dx$ as the difference in the cost of a 25-meter cable and a 30-meter cable. Recognizing this expression as an illustration of the FTC helped students give an accurate statement. Students were required to use correct units of meters (some slipped and wrote feet) and dollars (not price, cost, or money).

Part (c): Students were asked to write an expression for the profit of a cable k meters long. This problem required the same understanding required in part (a). There was difficulty in accurately using a variable k in the expression $P(k) = 120(k) - \int_0^k 6\sqrt{x} dx$ dollars. Again, students had to recognize profit as the difference in revenue and cost, and that the integral was evaluated with respect to x on an interval of $0 \leq x \leq k$. This portion of AB 3 was worth two points. The first point was given for a correct definite integral using $6\sqrt{x}$, and the second point was granted for the correct profit statement as in part (a).

Part (d): The final question for AB 3, determining the maximum profit for the cable company, was worth four points. Students who were successful in part (c) had a better chance at the first few points of this problem. Of course, it was required to see the statement $P'(k) = 0$ for the first point. Using the second part of the FTC on the statement in part (c) and solving $120 - 6\sqrt{k} = 0 \Rightarrow k = 400$ earned the second point. Students were not given the second point if $k = 400$ was not supported with an algebraic expression. (A

graphical determination was not accepted.) Students then had to find the profit: $P(400) = \$16,000$ for the third point. A calculator could be used for the evaluation of $P(400)$, including the integral portion of the profit statement. The fourth and final point was awarded for the justification of finding the absolute maximum profit at $k = 400$. An interval chart or number line for $P'(x)$ and $P(x)$ were incorrectly used by many students. Students had several options for demonstrating that this is the absolute maximum profit. Students had to state that $k = 400$ is the only critical point and that $P(x)$ increases and then decreases at $k = 400$, or $P'(x)$ changes from positive to negative at $k = 400$, thus justifying the answer globally. A local argument failed to earn the fourth point.

Interpretation and understanding are essential to the training we provide students in these advanced placement courses. Students continue to need practice handling more than the algebraic methods of using the FTC. What do these integral statements mean? What units are associated with each component of the statement? Can students apply a statement about a relationship, such as profit, for particular as well as general situations? As AP Calculus teachers we must be resourceful and take the time to find and ask the good questions to help our students become comfortable and precise when speaking and writing about mathematics, reducing ambiguities and inaccuracies.

AB 4 - The Area-Volume Problem

Larry Peterson - Northridge High School, Layton, UT

AB 4 was the area-volume problem. For the first time in many years it appeared on the non-calculator portion of the exam. Students were given two simple functions, $y = 2x$ and $y = x^2$, that enclosed a region in the first quadrant. Part (a) asked the student to find the area of the region. Parts (b) and (c) asked the students about the volume of two solids with different cross-sections, one perpendicular to the x -axis and one perpendicular to the y -axis. In keeping with the Test Development Committee's philosophy, there was an easy part, a moderately difficult part, and a challenging part to the question.

Part (a): The vast majority of students got this part of the question correct. This part of the problem was awarded three points: the first point was given for setting up the correct integral expression with or without correct limits (but some type of limits was required); the second point was earned for writing the correct antiderivative; the final point was earned for evaluating the anti-derivative correctly. Common mistakes included using an interval from $x = 1$ to $x = 2$ or from $x = 0$ to $x = 4$, misreading the problem to find the volume of a solid of revolution, or reversing the order of the functions. A reversed integrand yielded a value of $-\frac{4}{3}$ which did not

earn the answer point. However, a student could recover from this error by explicitly indicating the area was $\frac{4}{3}$. This is a typical

convention for correcting sign errors. As in previous readings, writing the incorrect statement $-\frac{4}{3} = \frac{4}{3}$ did not gain the point.

Students cannot correct errors with additional errors. A few students worked the problem using dy slicing. Correct solutions were scored in a similar manner.

Part (b): This part of the question asked the student to find the volume of a solid with a known cross-section. The function for the area of the cross-section was given as $A = \sin\left(\frac{\pi}{2}x\right)$, which should have made the problem straightforward for students. Again, as in

Part (a), the first point was earned for writing the correct integral; the second point was given for giving the correct antiderivative, and the third point came from the correct evaluation. Unfortunately, many students forgot that the volume of a solid with a known cross-

section can be found by $\int A(x)dx$. Many students tried to make some substitution such as $\int (2x - x^2)^2 dx$ or $\int \left(\sin\left(\frac{\pi}{2}x\right)\right)^2 dx$ or

$\int \sin(2x - x^2)dx$. Any integrand that was not in the form $k \int \sin\left(\frac{\pi}{2}x\right)dx$ made students ineligible for any points on this section.

Many students had a difficult time using u -substitution correctly for $\left(\frac{\pi}{2}x\right)$. To be eligible for the antiderivative point and the

evaluation point, the antiderivative had to be in the form $k \cos\left(\frac{\pi}{2}x\right); k < 0$.

Part (c): This part asked students to find the volume of a solid whose cross-section was a square drawn perpendicular to the y -axis. The first two points were given for the correct integrand and the third point came from correct limits of integration, 0 to 4, regardless of the integrand. Each term of the integrand was worth one point. Errors in the use of the square for the integrand were deducted from the two integrand points. Unfortunately, this part graded quite quickly since many students failed to recognize they needed to

write the functions as inverses in y and to use y limits from 0 to 4. Few students earned points for their work because it was incorrect. Students who wrote a correct integrand for the volume of squares perpendicular to the x -axis as $\int_0^2 (2x - x^2)^2 dx$ earned one point as a special case.

Recommendations: Students should be experienced in u -substitution for integrals. This type of question often appears on the multiple choice portion of the exam as well. It is also clear that students should be able to write simple functions as $y = f(x)$ and $x = f(y)$ while recognizing the correct limits. Finding the volume of a solid with a known cross-section has been a staple on the exams for the last ten years. It is important that students understand the concepts of setting up the definite integral as well as the mechanics in solving them.

AB 5 / BC 5 - Show Me the Calculus **Virge Cornelius - Lafayette High School, Oxford, MS**

This past reading marked my tenth year. It was my fourth year to serve as a Table Leader and my third year to assist with a question. As usual, the whole experience was incredibly powerful, particularly for those of us in small towns who may be the only "advanced math" teacher for miles around. Even though I left my husband and two young children at home for 12 days, I actually had tears in my eyes as the charter buses pulled away from the hotel to take us to the airport.

Before we arrived in Kansas City to work on AB 5 / BC 5 during the pre-reading, this question's working name was "Tabular Data." But, as we spent more and more time preparing to brief the readers, we affectionately renamed this question the "Show Me" problem, named both for our location in Missouri (The Show Me State) and for the fact that in all parts (a-d), students are asked to SHOW THE WORK. As AP teachers, we need to remember to always expect our students not only to arrive at the correct answers, but also to clearly show the steps that lead to their conclusions.

In part (a), students were asked to estimate $f'(4)$, showing the work that led to their answers. This entire part was worth one point.

The correct answer, -3 , without a difference quotient, earned no points. The difference quotient $\frac{f(5) - f(3)}{5 - 3}$ without the correct answer earned no points. However, $\frac{-2 - 4}{5 - 3}$ alone earned the point, assuming the student did not make a subsequent simplification error. Note: Teachers should encourage certain students to exercise caution when simplifying on the non-calculator portion of the exam!

In part (b), students were asked to evaluate $\int_2^{13} (3 - 5f'(x))dx$. This part was worth two points, one point for the use of the fundamental theorem and the second for the answer. We looked to see if the students were able to come up with $f(13) - f(2)$ to award the fundamental theorem point. Correctly handling the 3 (either geometrically or analytically), multiplying by -5 , and pulling the correct values from the table all went into the answer point.

In part (c), students were asked to use a left Riemann sum with sub-intervals indicated by the data to approximate $\int_2^{13} f(x)dx$. This part was worth two points. The first point was for showing the left Riemann sum; the second point for the answer. To award the first point, readers needed to see both plus and times signs. In other words, $1 + 8 - 6 + 15$ did not earn the Riemann sum point, however $1(1) + 4(2) + (-2)(3) + 3(5)$ earned both the first and the second point (since simplification is unnecessary). Pulling the correct values from the table went towards the answer point.

In part (d), students were asked to use the tangent (and secant) line at $x = 5$ to show that the actual value of $f(7)$ is less than (or more than) the estimated value. This part was worth four points. The first point was for use of a tangent line, the second point for showing that the value of the tangent line at $x = 7$ (which is 4) is greater than $f(7)$. Many students wrote a fine equation of a tangent line whereas some students successfully explained, in words and numbers, what the result of a tangent line would be. Both of these methods earned the first point. The student would then need to evaluate the tangent line at $x = 7$ and explain that since $f''(x) < 0$ (which means $f(x)$ is concave down - we took either), the tangent line overestimates $f(7)$. The third and fourth points followed the same logic, except with respect to the secant line. Students who were exclusively formulaic and never appealed to the point $(5, -2)$ had a difficult time earning any points.

In all, AB 5 / BC 5 graded quite smoothly and fairly once the readers got into it. Most feedback from the readers was positive, whether anecdotal or on the optional feedback forms. Tips for teachers: 1. Students need to be warned to be careful with their algebraic and arithmetic manipulations (parts a, b, c, d). 2. Students need more practice writing mathematics (part d). 3. Students need to know how to compute and use Riemann sums (part c) and need to be familiar with the consequences of concavity (part d).

AB 6 – Graph Analysis Problem

John Epler - Buckhannon-Upshur High School, Buckhannon, WV

Part (a): Students are asked to find all values of x at which f would have a point of inflection. Based on the graph of f , there are two values that satisfy: $x = -2$ and $x = 0$. The reason is that at both of these points, the graph of f changes from increasing to decreasing or vice versa. There were two points available for this part: one for the correct identification of either value of x and one for the correct justification along with both correct values of x .

Many students correctly identified one or both of the values for x . If students did not identify both values, they were ineligible for the justification point. Of the students who did identify both values, many did not give the correct justification of why they were the points of inflection. Students were not given this point if they appealed to the concavity of f based on f'' or by stating that f' changed directions. Acceptable justification for this problem ranged from correctly giving a f' or f'' argument such as: f' changed from decreasing to increasing at $x = -2$ and from increasing to decreasing at $x = 0$, f' has a minimum value at $x = -2$ and a maximum value at $x = 0$, f'' changes from negative to positive at $x = -2$ and from positive to negative at $x = 0$, f'' changes sign at $x = -2$ and at $x = 0$, or f'' crosses the x -axis at $x = -2$ and at $x = 0$.

Part (b): Students are asked to find $f(-4)$ and $f(4)$. In order to find $f(-4)$, students must correctly calculate the area under the graph of f' from 0 to -4 . This portion of the graph is given as a semicircle, so students should use this area and then subtract the area

of a rectangle that would enclose this semicircle. The area of the rectangle is 8 square units and the area of the semicircle is $\frac{4\pi}{2} = 2\pi$.

Students were also given that $f(0) = 5$, therefore $f(-4) = 5 - (8 - 2\pi) = 2\pi - 3$. There were two points available for this part: one for correctly expressing the integral from -4 to 0 and one for correctly calculating the value of $f(-4)$. In order to find $f(4)$, students

had to correctly calculate the area under the graph of f' from 0 to 4. This portion of the graph is given by the expression $5e^{\frac{x}{3}} - 3$ and

students were to use again the fact that $f(0) = 5$. Therefore, $f(4) = 8 - 15e^{\frac{4}{3}}$. There were three points available for this part: one for correctly expressing the integral from 0 to 4, one for the correct antiderivative, and one for correctly calculating the value of $f(4)$.

Many students correctly expressed one or both of the integrals in the part. Acceptable expressions were given as: $f(4) = \int_0^4 g(x)dx$ or

$f(4) = -\int_4^0 g(x)dx$ or seeing the expression $8 - 2\pi$ and $f(4) = \int_0^4 \left(5e^{\frac{x}{3}} - 3\right) dx$. Some students tried to generate the expression for the

semicircle, which made finding the integral very difficult. Many students were able to give one of the expressions for $f(-4)$ but forgot to use the initial condition that was given in the statement of the problem. Once again, many students were able to give the integral

expression $f(4) = \int_0^4 \left(5e^{\frac{x}{3}} - 3\right) dx$ but not give the correct antiderivative. Many incorrectly multiplied by $1/3$ rather than dividing by

this fraction. Many again did not use the initial condition for this part or incorrectly evaluated the definite integral (most when they substituted $x = 0$ in assumed it would equal 0 rather than 15).

Part (c): Students were asked to find the value of x for which f has an absolute maximum. For students to find this value, they must use the x -intercepts and interpret the graph of f' as being above or below the x -axis. There are two possible values of x : -2 and

$3\ln\left(\frac{5}{3}\right)$. The only value that satisfies at the point where the graph transitions from $f'(x) > 0$ to $f'(x) < 0$ is $x = 3\ln\left(\frac{5}{3}\right)$. There

were two points available for this part: one point for correctly identifying the absolute maximum as $x = 3\ln\left(\frac{5}{3}\right)$ and one for the correct justification for this value.

Many students attempted to set $f' = 0$ and then solve for x , rather than use the value that was given in the problem. Once a value was reached, students used values obtained in part (b) to help justify that $x = 3 \ln\left(\frac{5}{3}\right)$ was the absolute maximum and rule out the endpoints of the closed interval. Many students did not commit to the value of $x = 3 \ln\left(\frac{5}{3}\right)$ as the absolute maximum and were not eligible for the justification point. Students who used the values from part (b) were ineligible if they were incorrect in those values.

BC 3 – The Diver Problem

Christian Haich - Strake Jesuit College Preparatory, Houston, Texas

Comments on Scoring Part (a)

This part was worth 3 points. The first point was awarded for showing evidence of setting $\frac{dy}{dt} = 0$. This could have been done in a variety of ways, including writing " $\frac{dy}{dt}$ changes sign at ...", showing $t = \frac{3.6}{9.8}$, or many other ways. Simply showing $t = 0.367$ was not sufficient. The second point was awarded for showing evidence of solving the differential equation $\frac{dy}{dt} = 3.6 - 9.8t$ correctly. Because this is easy to do, no intermediate work was necessary. It was sufficient to show the solution $y(t) = 3.6t - 4.9t^2 + 11.4$. It was also sufficient to show $y(t) = 3.6t - 4.9t^2$ with $t = 0.367$ plugged in. The third point was awarded for correctly using the initial condition. In order to receive this point, students had to have a t value that they determined. Simply showing the correct answer to part (a) without a valid t value was not enough.

Comments on Scoring Part (b)

This part was worth 2 points. The first point was awarded for writing some equation equivalent to $11.4 + 3.6t - 4.9t^2 = 0$. This equation appeared in a wide variety of forms, including integral equations. The point could be earned by writing any equation of the form $3.6t - 4.9t^2 + C = 0$, but the coefficient of t^2 had to be -4.9 . Many student defined $y(t)$ in part (a) and simply wrote $y(t) = 0$, which earned the point. The second point was awarded for the correct answer, which could only happen by using the 11.4 meter initial condition. Any student who wrote $y(t) = 3.6t - 4.9t^2 + 11.4$ and $A = 1.936$ seconds without explicitly setting $y(t) = 0$ earned 1 out of 2 points for this part.

Comments on Scoring Part (c)

This part was worth 2 points. The first point was awarded for writing a definite integral with the correct integrand, a lower limit of 0, and any A value imported from part (b). If no A value was determined in part (b), a student could just write A . The integrand appeared in a variety of forms, since $\frac{dy}{dt}$ and $\frac{dx}{dt}$ were defined in the problem in the problem description. Even $\sqrt{dx^2 + dy^2}$ was accepted as a correct integrand. The second point was earned for the correct answer or an answer consistent with the student's answer from part (b). If the answer from (b) was incorrect, the consistent answer part (c) had to be greater than 11.4 meters.

Comments on Scoring Part (d)

This part was worth 2 points. The first point was awarded for evaluating $\frac{dy}{dt}$ and $\frac{dx}{dt}$ at $A = 1.936$ seconds (or the value that the student determined in part (b)) AND placing these values in a ratio. The student had to show evidence of determining a ratio of $\frac{dy}{dt}$ to $\frac{dx}{dt}$ or $\frac{dx}{dt}$ to $\frac{dy}{dt}$ AT a valid t value. Simply writing $\frac{dy}{dx} = -19.21913$ was not enough. The second point was awarded for the correct answer of $\tan^{-1}(19.21913)$. Many students wrote $\tan^{-1}(-19.21913)$, which does not give an angle in the correct range. Many students wrote $\tan^{-1}(-19.21913) = 1.518$, which is an incorrect statement. To earn the point, students had to determine the correct answer and show that they knew the angle was in $0 < \theta < \frac{\pi}{2}$. If students converted their angles to degrees, that earned the point if they labeled their answer as degrees.

General Comments

- Many students are not aware of or ignore the "3 numbers after the decimal point" rule. If students lost a point because of this on one part of this question, they were declared immune against further deductions for the same reason in this problem.
- It was evident that many students came back to this problem later without a calculator. Some students wrote things like, "If I had my calculator, I would ..." This is a waste of time, and mostly unnecessary. The student could express the answer in (a) in fraction form, could use the quadratic formula to express an answer in (b), and express the answer in (d) in the form \tan^{-1} (a valid fraction). The only part *requiring* the use of a calculator was part (c).
- Calculator syntax is not valid.
- Many students did not earn full credit because they did not show sufficient steps in parts (a), (b), and (d). Emphasize to students that they show a few small steps on paper, even if they can do it all in their head or on their calculators.
- Some students tried using their physics knowledge in this problem with varying degrees of success. If done properly, this approach is valid and earns full credit, but simple integration or anti-differentiation is simpler and gets you to the same place.
- Some students tried to express y as a function of x instead of as a function of t . Few were successful with this approach.

BC 4 – Differential Equation

Vicki Carter - West Florence High School, Florence, SC

In this problem, students were presented with a differential equation with an initial condition. It was the split problem on the BC exam; 6 points were part of the AB sub-score grade. The first two parts involved the BC topics of Euler's Method and a Taylor polynomial, while the third part was an AB-level separable, differential equation. It appeared in the non-calculator part of the exam. In part (a), students were instructed to use Euler's Method with two steps to approximate the value of a function. The students were given the initial condition at $x = -1$ and were to approximate $f(0)$. In part (b), the value of the 2nd derivative at $x = -1$ was given. Students were to find a 2nd degree Taylor polynomial about $x = -1$. In part (c), the students were asked to find the particular solution $y = f(x)$ for the differential equation with the initial condition $f(-1) = 2$.

Part (a) – 2 points

The first point was awarded for showing evidence of Euler's method with 2 steps using $\Delta x = 0.5$. In order to receive the "evidence" point, the student had to show the use of the initial condition $f(-1) = 2$, a value for $f(-.5)$ and $f(0)$, and the import of the value from the 1st iteration into the 2nd iteration. Students often presented the solution in a tabular format, both labeled and unlabeled. The 2nd point was the answer point. Arithmetic errors came off of the answer point. Readers accepted both $=$ and \approx in the presentation of the approximation. A minimal amount of work in the presence of the correct answer typically earned the student both points.

Part (b) – 1 point

The 2nd degree Taylor polynomial was either right or wrong. Students were allowed to import an incorrect but declared y' value from part (a). The students had to present a Taylor polynomial so the use of \dots or $+\dots$ after the polynomial would not earn the point. If the used " $y =$ " or " $f(x) =$ " the polynomial was accepted. Any error in simplification lost the point, even in the presence of a correct solution.

Part (c) - 6 points

The first point was the separation point. There were 3 categories of separation – good separation, bad separation, and no separation. The good separation also included the case where one integrand was incorrect by a multiple of -1 . This student lost the first point but was eligible for the remaining 5 points.

Some examples of bad separations are: $\int \frac{dy}{1-y} = \int 6x^2 dx$; $\int \frac{dy}{y} = \int x^2 dx$; $\int \frac{dy}{y} = \int (6x^2 - x^2) dx$. As long as the students retained a logarithm in the antiderivative, they were eligible for 4 of the 6 points (they lost the separation point and the answer point.) A student received 0/6 for no separation.

The 2nd and 3rd points were awarded for correct antiderivatives; one point for each antiderivative. The presentation of the absolute value signs in $\ln|6-y|$ was not necessary due to the given initial condition, but the absolute value signs were required for the presentation of any other natural log expression. The correct appearance of the constant of integration either in the step showing the antiderivatives or in a later step earned the fourth point. A late constant, one that appears as an addend after the exponentiation, did not earn the point. The use of the initial condition earned the fifth point. The student had to earn the 4th point, the constant point, to be eligible for the 5th point. If the constant of integration was introduced in the student's work, the initial condition could show up at various steps in the solution process. The 6th point was the presentation of the solution in the form of $y = f(x)$. In order to receive the solution point, the student had to earn the separation point (or have the multiple of -1 integrand), one of the antiderivative points

including either $\ln(6-y)$ or $\ln|y-6|$, the constant of integration point, and the use of the initial condition point. Any arithmetic or algebraic errors came off the answer point.

Summary

In part (a) we frequently saw the addition of $4 + \frac{1}{4} = \frac{5}{4}$. This is a case where a simple arithmetic error cost the students a point.

Teachers should encourage students to label any tabular work associated with Euler's method. It was often difficult to award any points for incorrect, unlabeled work which appeared in a table.

A common error in part (b) was the use of $(x-1)$ in the polynomial. Even though the value of $\frac{d^2y}{dx^2} = -12$ was given in the question, students sometimes dropped the negative sign in the polynomial. Another common error occurred when the student forgot to divide by 2 or (2!). Some students mistakenly presented just the 2nd degree term of the Taylor polynomial.

In part (c), we saw many sign errors as the students evaluated $\int(6-y)dy$ which caused the student to lose one of the antiderivative points. Other errors with negative signs occurred as the students worked through the problem. In most cases these errors came off the answer point. With this being a BC question, we did see occasional use of an integrating factor in the solution of the differential equation. We also saw several instances in which the students tied the initial condition to a separation involving definite integrals; i.e. the students solved $\int_2^y \frac{dy}{6-y} = \int_{-1}^x x^2 dx$. Overall, this was a good question for the students.

BC 6 – Infinite Series

Bob Angley - Charlotte Christian School, Charlotte, NC

Part (a) was worth two points. The first point was awarded for the correct presentation of the first four terms and the second point for a correct general term. Many students appeared confused by the given series being centered at $x = 0$ and being asked to present one centered at $x = 1$. Those students resorted to trying to find the first three derivatives of $f(x)$ and were quickly in trouble because of the need for the chain rule and the product rule in the second and third derivatives. Those students who observed the $(x-1)$ correctly gained entry into the problem easily. Many of those still made errors involving the laws of exponents. Many had the exponent on the fourth term as 5 or 8 rather than 6. Students who were awarded the first point found it relatively easy to come up with a correct general term, thus earning the second point. Students who made errors in the exponents usually missed the second point.

Part (b) was worth two points also. If the student had a correct answer in (a) he needed to have the correct answer in (b). If a student missed part (a) he needed an answer that was a Taylor series consistent with his answer in part (a). The problem was well constructed so that a student doing correct work would get an easily manageable answer in part (b). Unfortunately, many students did not handle the subtracting of 1 from both sides of the equation and dividing by $(x-1)^2$ well. They subtracted one from each term of the expression and divided each term by $(x-1)^2$. Students making this error no longer had a Taylor series and were ineligible for either point.

Part (c) was worth three points. The student was asked to use the ratio test to find the interval of convergence for the Taylor series found in part (b). The first point was for having a correct ratio of two consecutive terms (n th and $(n+1)$ th or their equivalent) using the student's general term from part (b). This was a soft point where graders read with the student as long as the student had a general term in part (b). The requirement for the general term to be that of a Taylor series was waived for this point. Since many students had

expressions in the form $\frac{(x-1)^n - 1}{(x-1)^2}$ the algebra in their ratio was tedious. Students who had an expression containing $2n$ in their general term lost this point for writing $2n+1$ rather than $2(n+1)$ or its equivalents. To be eligible for the last two points in part (c) the student needed the correct general term in part (b) or one in the form $\frac{(x-1)^{2n}}{n!}$. To earn the second point the student needed to

evaluate the limit correctly and indicate that the value was less than 1. The students who made errors in this part frequently made an algebra error while simplifying the ratio or confused the fact that n was becoming infinitely large rather than x and came up with expressions like $|x-1| < 1$. When this happened the student was required to check for convergence at his endpoints in order to gain the third point. The third point was awarded for a correct interval of convergence. Almost all the students who received both of the first two points go this point also.

Part (d) was worth two points. The student was asked to use the Taylor series for f about $x = 1$ to determine if f had any points of inflection. The student could earn the first point by finding a correct second derivative containing either a correct summation expression for the second derivative or write out at least three correct terms to the second derivative and indicate somehow that it was an infinite series. For the students who had correct work in part (b) the work was manageable. Some of them lost this point for not indicating that the second derivative continued past the first three terms. Most students were in trouble in part (d) because their function in part (b) was a rational expression and finding the second derivative seemed like an impossible task. The second point came for showing that there were no points of inflection because the correct second derivative was 1 plus terms with positive coefficients that were raised to even powers. The emphasis was on the student showing the non-negativity of all the terms. Generally, the students who had correct work in part (b) made the proper connections to earn this point.

General Reflections on BC 6

- Most students seem to have a rather loose grip on series topics. I think most students know they can form other infinite series by substituting expressions into a given infinite series. However, since this series was to be centered at $x = 1$ when the given series was centered at $x = 0$, many students saw that as requiring that they use the definition of a Taylor series and compute the derivatives needed. The penalty for that mistake was severe here since the student did not have an easy re-entry to the problem in a later part.
- Under the pressure of the exam many students failed to realize that writing an expression like $(x-1)^2 - 1$ or $(x-1) - 1$ either re-centered the series or caused it not to be a Taylor series. The same thing is true for those who wrote quotients of polynomials in each term.
- The errors involving misuse of laws of exponents and the incorrect use of parentheses indicate a need for stressing good math fundamentals in all algebra classes.
- Series questions usually ask students to write a second or third degree Taylor polynomial when given the necessary derivatives or their values at $x = a$. This was present in BC-4 part (b) this year.

Advice for Teachers on BC 6

- Even though the term for term Maclaurin series for e^x was given this year to help the student make fewer substitution errors, students need to know the Maclaurin series expressions for e^x , $\sin x$, and $\cos x$.
- Students need to practice more examples that involve re-centering the series through the use of substitution.
- They also need examples of creating new series by simple algebraic manipulations as in Part (b).
- The Ratio Test and the Alternating Series Test need to be emphasized as they occur more frequently on the AP exam.
- Students need practice in evaluating limits when finding the radius of convergence for a power series.
- Students need to know that the AP exam is more about understanding concepts and applying the proper calculus technique than it is about algebraic manipulation of equations. If work seems to be increasingly tedious, the student needs to pause and rethink his options.
- The fundamental idea that an infinite series can be integrated or differentiated term by term to produce other infinite series is an important concept.

**2009 AB EXAMINATION
FREE-RESPONSE QUESTION FREQUENCY**

Prepared by Trish Morris

FUNCTIONS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Zeros | | | | | | | | | | |
| Asymptotes | | X | | | | | | | | |
| Symmetry | | | | | | | | | | |
| Domain | | | | X | | | | | | X |
| Odd/Even | | | | | | | | | | |
| Range | | | | | | | | | | X |
| Inverse | | | | | | | | | | |
| Limits | | X | | | | | | | | |
| Linear Equation | | | | | | | | | | |
| Continuity | | | X | | | | X | X | | |

DIFFERENTIAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Analyze a Function Given as a Table of Values | X | X | | X | X | | X | | | |
| Tangent Line Equation | X | X | X | X | X | | X | X | X | X |
| Differentiation & Evaluation | X | X | X | | | | X | X | X | |
| Increasing & Decreasing Functions | | X | X | | | X | X | X | X | |
| Critical Numbers, Maximum & Minimum Points – Relative & Absolute | X | X | X | | X | X | | X | X | X |
| Concavity | | X | X | | X | | X | X | X | |
| Inflection Points | | X | X | | X | | | | X | |
| Average Rate of Change | X | | | X | X | X | | | X | |

DIFFERENTIAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Critical Numbers, Maximum & Minimum Points, Increasing & Decreasing, Concavity, Inflection Points from the graph of $f'(x)$ | X | X | | X | | X | X | X | | X |
| Critical Numbers, Maximum & Minimum Points, Increasing & Decreasing, Concavity, Inflection Points from a table of values of $f(x)$, $f'(x)$, & $f''(x)$ | | X | | | X | | | X | | |
| Curve sketching or analyzing data using information from a table of values of $f(x)$, $f'(x)$, & $f''(x)$ or from the graph of $f'(x)$ | X | X | | X | X | | | X | | |
| Mean Value Theorem for Derivatives | | X | X | | X | | | X | | |
| Implicit Differentiation | | | | | | X | | | | X |
| Linear Approximation | X | | X | | X | | | X | X | |
| Related Rates | | X | X | | | | X | X | | |

INTEGRAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Area and/or Interpretation | X | X | X | X | X | X | X | X | X | X |
| Riemann Sums using Left, Right, Midpoint Evaluation Points | X | | X | X | | | X | | | |
| Properties of Integrals | X | X | | | | | X | X | X | |
| Trapezoidal Rule/Approximation | | X | | | X | | | | X | |
| Fundamental Theorem of Calculus | X | X | X | | X | | X | X | X | |
| Mean (Average) Value | X | | | X | X | X | X | | X | |
| Volumes of Solids: Disks and Washers | | | X | X | X | X | X | X | X | X |
| Volumes with known Cross Sections | X | X | X | | | X | X | | | X |

INTEGRAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Solving Differential Equations:
Separation of Variables | | X | | X | X | | X | | X | X |
| Drawing Slope Field from Differential Equation | | X | | X | X | X | | | | |
| Rectilinear Motion: Equation(s) for Position, Velocity, &
Acceleration; Direction of Motion; Total Distance | X | | X | X | X | X | X | X | | X |
| Rectilinear Motion: Position, Velocity, & Acceleration;
Total Distance from
the Graph of Velocity | X | X | | | X | | | | | X |
| Rectilinear Motion Analysis from a Graph | X | X | | | | | | | X | |
| Definite Integral as an Accumulator | X | X | X | X | X | X | X | X | | X |
| Accumulation of the Derivative with
Initial Condition | X | X | X | X | X | X | X | | | |

CALCULATOR

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Draw a Graph in a Given Window | | | X | | X | | | X | | |
| Find the Zeros of a Function | | | | | | | | X | | |
| Find the Intersection Points of Two Graphs | | X | | X | X | X | X | | X | X |
| Evaluate a Definite Integral | X | X | X | X | X | X | X | X | | X |
| Evaluate a Derivative | X | | | | X | X | X | X | X | |

**2009 BC EXAMINATION
FREE-RESPONSE QUESTION FREQUENCY**

Prepared by Trish Morris

FUNCTIONS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Domain & Range | | X | | | | | | | | X |
| Odd/Even | | | | | | | | | | |
| Intercepts | | | | | | | | | | |

LIMITS & CONTINUITY

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|---------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Finite Limits | | | X | | | | | | X | |
| Limits at Infinity
Infinite Limits | | | | X | | | | | | X |
| Definition of
Continuity | | | | | | | | | | |

DIFFERENTIAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Analyze a Function Given as a Table of Values | X | X | | | X | | | | X | |
| Tangent Line Equation | X | X | X | | X | | X | | X | X |
| Differentiation & Evaluation | | X | | | | | | | X | |
| Increasing & Decreasing Functions | | X | X | | | | X | X | X | |
| Critical Numbers | X | X | X | X | | | | X | X | X |
| Concavity | | X | X | | X | | X | X | X | |
| Inflection Points | X | X | | | | | X | | | |
| Average Rate of Change | X | | | X | X | X | | X | | |
| Extreme Values | X | X | X | X | X | | | X | X | |

DIFFERENTIAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Higher Order Derivatives | | | X | X | | | | | | |
| Optimization Problems: Maximum & Minimum; Relative and Absolute | | | | X | X | | | X | | |
| Curve Sketching or analyzing data using information from a table of values of $f(x)$, $f'(x)$, & $f''(x)$ or from the graph of $f'(x)$ | X | X | | | X | | | X | X | |
| Mean Value Theorem for Derivatives | | X | | | X | | | | | |
| Implicit Differentiation | | | | X | X | X | | | | X |
| Linear Approximation | X | X | X | | | | | | X | |
| Related Rates | | | X | | | | X | | | |

INTEGRAL CALCULUS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Area and/or Interpretation | X | X | X | X | X | X | X | X | | X |
| Riemann Sums using Left, Right, & Midpoint Evaluation Points | X | | X | X | | | | | | |
| Properties of Integrals | X | X | | | | | X | X | | |
| Trapezoidal Rule/Approximation | | X | | | X | | | | X | |
| Fundamental Theorem of Calculus | X | X | | | X | | X | X | | |
| Mean (Average) Value | X | | | X | X | X | | X | X | |
| Volumes of Solids: Disks and Washers | | | X | X | X | X | X | X | | X |
| Volumes with known Cross Sections | | X | X | | | | X | | | X |
| Rectilinear Motion: Equations for Position, Velocity, & Acceleration; Direction of Motion; Total Distance | | | | | X | | | | | X |

INTEGRAL CALCULUS

| | <u>'09</u> | <u>'08</u> | <u>'07</u> | <u>'06</u> | <u>'05</u> | <u>'04</u> | <u>'03</u> | <u>'02</u> | <u>'01</u> | <u>'00</u> |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Rectilinear Motion: Position, Velocity, & Acceleration; Total Distance from the Graph of Velocity | X | X | | | X | | | | | X |
| Rectilinear Motion Analysis from a Graph | X | X | | | X | | | | X | X |
| Definite Integral as an Accumulator | X | X | X | X | X | X | X | X | | X |
| Arc Length, Surface Area, and Work | | | | | | | | | | |

CALCULATOR

| | <u>'09</u> | <u>'08</u> | <u>'07</u> | <u>'06</u> | <u>'05</u> | <u>'04</u> | <u>'03</u> | <u>'02</u> | <u>'01</u> | <u>'00</u> |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Draw a Graph in a Given Window | | | X | | | | | | | |
| Find the Intersection Points of Two Graphs | | X | | X | X | | X | | | X |
| Evaluate a Definite Integral | X | X | X | X | X | | X | X | | X |
| Evaluate a Derivative | X | | | | | X | | X | X | |

METHODS OF INTEGRATION

| | <u>'09</u> | <u>'08</u> | <u>'07</u> | <u>'06</u> | <u>'05</u> | <u>'04</u> | <u>'03</u> | <u>'02</u> | <u>'01</u> | <u>'00</u> |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Integration by Parts | | X | X | | | | | | | |
| Integration by Partial Fractions | | | | | | | | | | |
| Improper Integrals | | | | X | | | | | X | |

DIFFERENTIAL EQUATIONS

| | <u>'09</u> | <u>'08</u> | <u>'07</u> | <u>'06</u> | <u>'05</u> | <u>'04</u> | <u>'03</u> | <u>'02</u> | <u>'01</u> | <u>'00</u> |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Solving Differential Equations by Separation of Variables | X | X | | X | | | X | | X | X |
| Logistic Differential Equations | | X | | | | X | | | | |
| Slope Fields | | X | | | X | | | X | | X |
| Euler's Method | X | X | | X | X | | | X | X | |

SEQUENCES AND SERIES

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Tests for Convergence | | | | | X | | | | | |
| Geometric Series | | | | | | | | | X | |
| Alternating Series & Error Approximation | | | X | | | | X | | | X |
| p-Series | | | | | | | | | | |
| Manipulation of a Power Series | X | | | X | | X | X | X | X | |
| Power Series: Radius of Convergence | | | | | | | | | | X |
| Power Series: Interval of Convergence | X | | | X | X | | | X | X | |
| Maclaurin Series | | X | X | | | | X | X | | |
| Taylor Series | X | X | X | X | X | X | | X | | X |
| Lagrange Error Bound | | X | | | | X | | | | |

PARAMETRIC, POLAR, AND VECTOR FUNCTIONS

| | '09 | '08 | '07 | '06 | '05 | '04 | '03 | '02 | '01 | '00 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Parametrically Defined Curves | | | | | | X | X | X | X | |
| Derivatives of Vector & Parametrically Defined Curves | X | | | X | | X | X | X | X | X |
| Tangent Lines to Parametrically Defined Curves | | | | | | X | X | | X | X |
| Arc Length Including Parametrically Defined Curves | X | | | | | | | | | |
| Velocity, & Acceleration Vectors for Motion on a Curve | X | | | X | | X | | X | | X |
| Polar Coordinate Graphs | | | X | | X | | X | | | |
| Area of Polar Curves | | | X | | X | | X | | | |

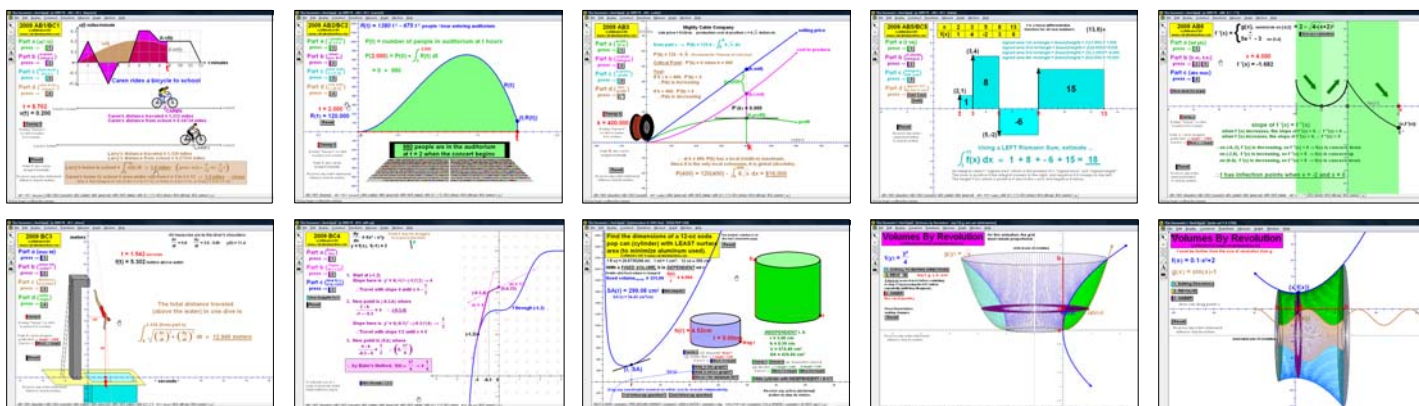
JUNE 2009 UPDATE AVAILABLE FOR *CALCULUS IN MOTION*

It is the month of June,
The month of leaves and roses,
When pleasant sights salute the eyes,
And pleasant scents the noses."
~ Nathaniel Parker Willis

AND... for Calculus In Motion,
The month of new files' issue,
When FRs come to life on screens,
And likewise in brain tissue."
~ Audrey Weeks

Yes, June is the month that heralds the release of new *Calculus In Motion* animations to incorporate into next year's lessons. This Year's update features not only animations to the 2009 AB & BC Calculus Exam Free Response Questions, but also includes many new optimization applications as well as additions to the volumes of solids of revolution (you now can revolve a region bounded by any two functions across any vertical or any horizontal axis of revolution). To get the update, send an email to Audrey Weeks at amweeks@aol.com with your serial number from the front of your *Calculus In Motion* CD along with your name/school information. The files either will be attached back to you free of charge (if you are within 2 years of the purchase of your CD or a prior update) or you'll receive the update order form (after 2 years, there is nominal fee).

If you're not already a user of *Calculus In Motion*, wait no longer! This collection of 161 interactive animations spans the entire year of calculus – graphing, limits, derivatives, integrals, theorems, related rates, optimization, areas, volumes, Newton's Method, slope fields, Euler's Method, Maclaurin / Taylor series polynomials, and more! Simply visit www.calculusinmotion.com (or email Audrey Weeks at amweeks@aol.com), link to "Order Form" and email, fax, or send your order ASAP so you'll be able to begin using the files with your students from week 1 of the upcoming school year.



Upcoming Conferences

T³ Conference (Teachers Teaching with Technology) March 6 - 7, 2010 Atlanta, Georgia

Special *conference within a conference* on Issues, Ideas, Innovations for technology in teaching calculus. This should be of special interest to AP Calculus teachers. The T³ conference is March 5 - 7 and the conference within a conference would be at most Saturday/half day Sunday on March 6 - 7. Contact Tom Dick (tpdick@math.oregonstate.edu) if you have a special interest in presenting at this conference.

NCTM

San Diego
April 21-24, 2010
www.nctm.org

NCCTM

Greensboro, NC
October 29-30, 2009

NCTM Regional

Nashville, TN
November 18-20

Items of Interest

NCTM is sponsoring **Fall 2009 E-Workshops**. Grade 9-12 sessions are entitled Implementing the Algebra Standard. Register at www.nctm.org/eworkshops/

My **website** offers many chapters, resources, etc for AP Calculus as well as other topics which may be useful in high school math courses. The link is <http://www.jackmathsolutions.com/> Several chapters of my AP calculus text are available on the website.

Jack Koenka, Math Consultant, Ryerson University, Toronto